# Mathematics or moonshine

non-Euclidean geometry in *The Monist* at the turn of the twentieth century

APMP 2020 Jemma Lorenat, Pitzer College jlorenat@pitzer.edu And to return to America, is not the *Monist* published at Chicago, that review which even to us seems bold and yet which finds readers?

> Henri Poincaré (1905) Author's Preface to Science and Hypothesis translated by George Bruce Halsted

Moreover, Dr. Halsted gives regularly each year a review of all productions relative to the non-Euclidean geometry, and he has about him a public deeply interested his work. He has initiated this public into the ideas of Hilbert, and he has even written an elementary treatise on "Rational Geometry," based on the principles of the renowned German savant.

> Henri Poincaré (1905) Author's Preface to Science and Hypothesis translated by George Bruce Halsted

To introduce this principle into teaching is surely this time to burn all bridges of reliance upon sensory intuition, and this is, I confess, a boldness which seems to me almost rashness.

The American public is therefore much better prepared than has been thought for investigating the origin of the notion of space.

> Henri Poincaré (1905) Author's Preface to Science and Hypothsis translated by George Bruce Halsted

As far as Professor Halsted, I believe he is quite pugnacious. But is he quite sane? In any case I hardly consider "Non-Euclidean Geometry" as mathematics at all. It seems so much moonshine.

> Alan Hawkesworth (1909) Letter to Paul Carus

- 1. prelude: "The Circle Squarer"
- 2. mathematics and *The Monist*
- 3. institutions and individuals
- 4. authority and dissent
- 5. words and meanings
- 6. agnotology and incommensurability

non-Euclidean geometry in *The Monist* at the turn of the twentieth century

- 1. prelude: "The Circle Squarer"
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life imitating art imitating life

- 1. prelude: "The Circle Squarer"
  - A. the race of circlesquarers
  - B. a work of fiction
  - C. a work of criticism
  - D. mathematical reception
  - E. rebuttal

But the race of circle-squarers, unmindful of the verdict of mathematics, that most infallible of arbiters, will never die out so long as ignorance and the thirst for glory shall be united.

Hermann Schubert (1891)

The Squaring of the Circle

A. the race of circlesquarers

## B. a work of fiction

- C. a work of criticism
- D. mathematical reception
- E. rebuttal

As to my story "The Circle Squarer," it is aimed at real, regular circle squarers with whom I had the pleasure of being officially bored in my capacity as editor. We have several of them on hand, and the idea of sketching this type of human beings has been in my mind for several years, — ever since we published Hermann Schubert's "Squaring of the Circle" in the second number of The Monist.

Paul Carus (1894) Letter to Charles S. Peirce

A. the race of circlesquarers

### B. a work of fiction

- C. a work of criticism
- D. mathematical reception
- E. rebuttal



Squarer" sent me a few days ago. It is a most faithful portrayal of the history of one form of primary delusional insanity, which ought to be a warning to those who ride hobbies, to dismount before it is too late. The alienist himself is very apt to be a hobby rider. I thank you for putting me on my guard.

Your visit among us was highly appreciated by the members of the Medical Staff and will be long remembered. We selfishly wish for many more such opportunities for gaining breadth.

A letter from Dr.Meyers just received. He states that he is on his way from Sault Ste.Marie to Buffalo, at which place he expects to stay for some time. I am Doctor

Yours respectfully,

hund Stearno

- 1. prelude: "The Circle Squarer"
  - A. the race of circlesquarers

### B. a work of fiction

- C. a work of criticism
- D. mathematical reception
- E. rebuttal

The Professor gazed at his visitor in utter dismay. "I am glad to see," continued Mr. Gorner, "that you are not so bigoted as your colleagues who would even refuse to listen to a man who has spent thousands upon thousands in the interest of science."

"I suppose you have had many sad experiences with mathematicians," continued the Professor sarcastically. "Undoubtedly you have found them altogether too dogmatic for your advanced views."

"Experiences?" cried Mr. Gorner, "Indeed I have had enough; but I will shame them all and when you publish my solution, they will regret having rejected so honorable an offer!"

- A. the race of circlesquarers
- B. a work of fiction
- C. a work of criticism
- D. mathematical reception
- E. rebuttal

I make bold to confess a growing measure of misgiving as to certain geometrical results that have played and are still playing a conspicuous rôle in the mathematics of the present epoch. I refer to the so-called non-Euclidean geometry, and I propose to utter a little note of protest or rather of question.

**Francis C. Russell (1909)** A Modern Zeno

- 1. prelude: "The Circle Squarer"
  - A. the race of circlesquarers

### B. a work of fiction

- C. a work of criticism
- D. mathematical reception
- E. rebuttal

The next morning Mr. Gorner went to the library and ordered all the books he could find in the catalogue on the number  $\pi$  and the quadrature of the circle. He soon felt his inability to comprehend the formulas and deductions, but remained, nevertheless, unshaken in his conviction that he was in possession of the truth.

- A. the race of circlesquarers
- B. a work of fiction
- C. a work of criticism
- D. mathematical reception
- E. rebuttal

These experts have wrapped up the doctrine in what to many is a maze of analytical language that requires a good deal of analytical erudition to compass and thoroughly possess. It makes the logician inclined to ask if these analysts have not mistaken some mere grammatical collocation of their analytical language for a real ideal possibility.

**Francis C. Russell (1909)** A Modern Zeno

- 1. prelude: "The Circle Squarer"
  - A. the race of circlesquarers

### B. a work of fiction

- C. a work of criticism
- D. mathematical reception
- E. rebuttal

"But, my dear sir," replied Mr. Gorner, "I know what I propose; and, having devoted my whole life to the problem, I ought to know better than you. I do not dispute that you know more about curves of the third and fourth order than I; so do not envy me my claim of understanding better the quadrature of the circle."

- A. the race of circlesquarers
- B. a work of fiction
- C. a work of criticism
- D. mathematical reception
- E. rebuttal

Considering that the modern man, aye, even the modern of so-called liberal education, finds himself lost between a disposition on his part to respect the august majesty of mathematics and a disposition tempting him to regard the non-Euclidean something as the mathematician regards the circle-squarer and the perpetual motions, it would seem as though it might be worth the while for some one of the non-Euclidean sect to so explain their doctrine as to make it manifestly clear and sound to minds as unable as mine.

Francis C. Russell (1909)

A Modern Zeno

- A. the race of circlesquarers
- B. a work of fiction
- C. a work of criticism

# D. mathematical reception

E. rebuttal

OR GEORGE BRUCE HALSTED B D' B GREELEY, COLORADO c' D c He applies 25 to prove that two straight lines which are parallel to a third in opposite senses are themselves porallel: 6'b 11 AH (to the right) and BB' 11 AH' (to the left). "Merefore bib is proved to be not perallel to B'B. [Had he been ] What shell we do about it? Shall 9 write a very brief article entitled " Eary non-Euclid". Yours always George Bruce Halsted

- A. the race of circlesquarers
- B. a work of fiction
- C. a work of criticism

# D. mathematical reception

E. rebuttal

Mr. Russell, the writer of the article, has evidently made some study of Non-Euclidean Geometry, especially of the writings of Lobatchevsky. But truly "a little learning is a dangerous thing." His study has been superficial and without understanding. It is not my intention to criticize the article in detail, but to point out two errors that make it almost worthless.

#### W. H. Bussey (1909)

Some Remarks on Mr. Russell's Article

- A. the race of circlesquarers
- B. a work of fiction
- C. a work of criticism

# D. mathematical reception

E. rebuttal

In a word, Russell has substituted a different axiom in place of the postulate of Euclid. His endeavor and achievement have however left nothing that could make a step towards disarming the pan-geometricians. We stand uninjured on the same ground as before in spite of all the desperate assaults from the strong hand of Russell, who has utterly failed to disground us.

Yoshio Mikami (1910) A Remark on F. C. Russell's Theorem

- A. the race of circlesquarers
- B. a work of fiction
- C. a work of criticism
- D. mathematical reception
- E. rebuttal

Of course professional expertness is an index of intellectual quality, but if other things be equal (an important condition truly) how is the professional expert better fitted to see more lucidly in dealing with the elements of geometry than any other person of good geometric faculty?

Francis C. Russell (1909) Mr. F. C. Russell Still Demurs

- A. the race of circlesquarers
- B. a work of fiction
- C. a work of criticism
- D. mathematical reception
- E. rebuttal



#### 2. mathematics and *The Monist*

- 3. institutions and individuals
- 4. Authority and dissent
- 5. Words and meanings
- 6. Agnotology and incommensurability

Hegeler, Carus, Schubert, and other mathematics in *The Monist* 

- 2. mathematics and *The Monist* 
  - A. editorial oversight
  - B. six years of Schubert
  - C. contents



Edward Hegeler (1835–1910)





#### Paul Carus (1852—1919)

Hegeler estate in La Salle, Illinois

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B. six years of Schubert

#### C. contents

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#### A. editorial oversight

- B. six years of Schubert
- C. contents

I am not a mathematician proper although I have always taken an interest in mathematics, and from my childhood I considered mathematics not as work but as a relaxation. At school I could do my mathematical work always without labor, and it was the only subject in which I could fearlessly go into any examination.

Paul Carus (1909) Letter to Alan S. Hawkesworth

#### A. editorial oversight

- B. six years of Schubert
- C. contents

My professor was the famous Herman Grassman [sic], of Stettin, whose name you may know of. I have written on mathematics repeatedly in former publications of mine [...] you will see my method of dealing with mathematical problems, which is always geometrical. I think mathematics more with my eyes than in the logical way. I want to make my thoughts visible as space relations.

Paul Carus (1909) Letter to Alan S. Hawkesworth

- 2. mathematics and *The Monist* 
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## Hermann Schubert (1848 – 1911)

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### A. editorial oversight

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To clear up such ideas and to correct the wrong impressions of cultured people who have not a technical mathematical training, is the purpose of the following pages.

Hermann Schubert (1893) The Fourth Dimension: mathematical and spiritualistic

A. editorial oversight

#### B. six years of Schubert

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mathematics in the United States at the turn of the twentieth century

# A. a research community

- B. journals and universities
- C. Chicago
- D. non-Euclidean geometry in *The Monist*

The historical conjunction during the last quarter of the nineteenth century of these various innovations and their innovators, of changed attitudes as to the value and desirability of research at both an individual and institutional level, and of the existence of a critical mass numbering some 1000 practitioners had produced, by 1900, a firmly grounded mathematical research community in the United States.

## Karen Parshall and David Rowe (1994)

The Emergence of the American Mathematical Research Community

- A. a research community
- B. journals and universities
- C. Chicago
- D. non-Euclidean geometry in The Monist

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THOMAS SCOTT FISKE

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- A. a research community
- B. journals and universities

### C. Chicago

D. non-Euclidean geometry in *The Monist* 



Papers Published by the American Mathematical Society.—Vol. I.

#### MATHEMATICAL PAPERS

READ AT THE

INTERNATIONAL MATHEMATICAL CONGRESS

HELD IN CONNECTION WITH THE

WORLD'S COLUMBIAN EXPOSITION CHICAGO 1893

EDITED BY THE COMMITTEE OF THE CONGRESS

OSKAR BOLZA

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#### UNIV. OF CALIFORNIA ATLOS ANDELES (JERAPY

NEW YORK MACMILLAN AND CO. FOR THE AMERICAN MATHEMATICAL SOCIETY 1896

102480

- A. a research community
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D. non-Euclidean geometry in *The Monist*  Professor Klein had attended the Congress as official commissioner from Germany, and had contributed much to the success of that gathering. [...] This by no means easy task he undertook solely from love of his chosen science, and in the hope that thus he might contribute to the promotion of research in America in those parts of pure mathematics which at present engage the attention of the foremost European investigators.

Henry S. White (1894) REVIEW: Klein's Evanston Lectures Bulletin of the American Mathematical Society
- A. a research community
- B. journals and universities

# C. Chicago

D. non-Euclidean geometry in *The Monist* 



#### George Bruce Halsted (1853-1922)

# Journal of Mathematics

#### PURE AND APPLIED.

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PUBLISHED UNDER THE AUSPICES OF THE JOHNS HOPKINS UNIVERSITY.

Πάντα γα μὸν τὰ γιγνωσκόμενα ἀριθμὸν ἐχοντι.—Philolaos.

Volume I. Number 3.

BALTIMORE: Printed for the Editors by John Murphy & Co.

B. WESTERMANN & Co., D. VAN NOSTRAND, TRÜENER & Co., London, GAUTHIER-VILLARS, Paris. 1878

#### BIBLIOGRAPHY OF HYPER-SPACE AND NON-EUCLIDEAN GEOMETRY.

BY GEORGE BRUCE HALSTED, Tutor in Princeton College, N. J.

UNTIL the present century the Euclidean Geometry was supposed to be the only possible form of Space-science; that is, the space analysed in Euclid's axioms was supposed to be the only non-contradictory sort of space. The Parallel-Postulate was generally supposed to be a consequence of the nature of straight lines, and demonstrable from the remaining postulates and axioms. The researches of Peyrard show that it was not given out by Euclid as an axiom, since in all the MSS. examined by him it is kept separate from the axioms, and has only been classed with them by an obvious error in modern times. The enormous number of unsatisfactory attempts to prove this postulate, led finally to a systematic development of the results obtainable when it is denied, and then sprang forth the Non-Euclidean or Absolute Geometry.

In the "Encyclopædie der Wissenschaften und Künste; von Ersch und Gruber; Leipzig, 1838; under "Parallel," Sohncke says that in mathematics there is nothing over which so much has been spoken, written and striven, as over the theory of parallels, and all, so far, (up to his time) without reaching a definite result and decision. He divides the attempts into three classes: — 1. In which is taken a new definition of parallels. 2. In which is taken a new axiom different from Euclid's. But just as Euclid's cannot be considered axiomatic, so is it with these new postulates. This led to the third, the largest and most desperate class of attempts, namely, to deduce the theory of parallels from reasonings about the nature of the straight line and plane angle. The article is followed by a carefully prepared list of ninetytwo authors on the subject, from the earliest times up to the year 1837.

In English an account of like attempts is given in the "Geometry without Axioms," by Perronet Thompson: Cambridge, 1833; where the author also makes an elaborate attempt of his own. These accounts may be considered to bring the subject up to the point where, through the perfectly original

- A. a research community
- B. journals and universities
- C. Chicago
- D. non-Euclidean geometry in *The Monist*

I have not published my Lecture before the Mathematical Congress, though Professor Tyler of Boston, and Prof. Ziwet, Editor of the Bulletin of the New York Mathematical Society, each asked to publish it in full.

George Bruce Halsted (1893)

Letter to Dr. Paul Carus

- A. a research community
- B. journals and universities
- C. Chicago
- D. non-Euclidean geometry in *The Monist*

Prof. Klein lent me his M. S. unpublished lectures on his favorite subject Non-Euclidean and Higher Geometry, and I saw that I could give him some "Salient Points", and I did so. From the Notes which Prof. Klein took on my lecture, and the questions he afterward asked me to be sure of their accuracy, I was satisfied that Chicago had contributed something to future "Vorlesungen" at Göttingen, and I was amply repaid.

# George Bruce Halsted (1893)

Letter to Dr. Paul Carus

- A. a research community
- B. journals and universities
- C. Chicago
- D. non-Euclidean geometry in *The Monist*

VOL. 4. No. 4.

# THE MONIST

#### A QUARTERLY MAGAZINE.

Editor: DR. PAUL CARUS.	Associates : { Edward C. Hegeler. MARY CARUS.
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CHICAGO:

THE OPEN COURT PUBLISHING COMPANY.

<sup>1894.</sup> 

- 1. prelude: "The Circle Squarer"
- 2. mathematics and *The Monist*
- 3. institutions and individuals

- 5. words and meanings
- 6. agnotology and incommensurability

old-fashioned and new-fangled geometers

#### A. consensus

- B. the mathematical "we"
- C. mathematical mysticism vs.philosophical perspective

I have not the conviction or the enthusiasm that would induce one to join the hyper-space crowd in their Quixotic chase after the Jack O'Lantern angle-sum less than two right angles, of the pangeometer Lobatschewsky. [...] My standpoint is that of the Euclidean geometer and the practical physicist.

**J. N. Lyle (1893)** Letter to Paul Carus

#### A. consensus

- B. the mathematical "we"
- C. mathematical mysticism vs.philosophical perspective

These considerations seem to me so obvious that I am surprised that it should be necessary to point them out. I am no scientist. I am a mere man-inthe-street, or rather — in the jungle. I am, however, an engineer by profession; and perhaps, as such, I have had more to do in the way of measuring spaces than falls to the lot of men in other professions.

# W. E. Ayton Wilkinson (1907)

The Logical Aspect of the Theory of Hyper-Spaces

#### A. consensus

- B. the mathematical "we"
- C. mathematical mysticism vs.philosophical perspective

In justice to mathematical science it must be added that the vast majority of theorists do not hold the above view. That the greatest among them should is perhaps to be construed as a confirmation of the good old doctrine that the gods are jealous of too great human success.

# W.B. Pitkin (1907)

A Logical Aspect of the Theories of Hyper-Spaces

#### A. consensus

- B. the mathematical "we"
- C. mathematical mysticism vs.philosophical perspective

The article of Mr. Chas. H. Chase on "Pseudo-Geometry" in the last issue of *The Monist* (pp. 465 —467) has most certainly been welcomed by all mathematicians, desirous of keeping our infallible science free from absurdities and chimeras. In fact it is so surprising how the fallacies of Lobatchevsky and Bolyai could find so many followers.

Francis Rust (1908) A Comment on Pseudo-Geometry

#### A. consensus

- B. the mathematical "we"
- C. mathematical mysticism vs.philosophical perspective

If Professor James or his pragmatist adherents speak of Euclid as superseded and no longer true, they are not to be taken seriously, and there is no need of refuting them; but the case is different when mathematicians of standing make similar declarations. [...] The problems concerning the foundations of geometry and of mathematics in general are by no means so definitely settled that one solution may be said to have acquired the consensus of the competent.

# Paul Carus (1909)

The Nature of Logical and Mathematical Thought

## A. consensus

#### B. the mathematical "we"

C. mathematical mysticism vs.philosophical perspective

We shall say nothing of the inept strictures of the Beoetians, whose coming Gauss predicted, and whose attitude determined him to reserve. But what shall we say to the acrid and captious criticisms to which Gauss, Riemann and their associates have been subjected by men of high standing in the scientific world.

# Ernst Mach (1904)

Space and Geometry from the Point of View of Physical Inquiry translated by T. J. McCormack

# A. consensus

# B. the mathematical "we"

C. mathematical mysticism vs.philosophical perspective

Have they never experienced in their own persons the truth that inquirers at the outermost boundaries of knowledge discover many things that do not slip smoothly into all heads, but which are not on that account nonsense?

# Ernst Mach (1904)

Space and Geometry from the Point of View of Physical Inquiry translated by T. J. McCormack

### A. consensus

#### B. the mathematical "we"

C. mathematical mysticism vs.philosophical perspective

A good paper on the subject or its philosophic import may be written by one who is not an expert mathematician, but it seems to me that such a paper should be carefully read by an expert mathematician before publication, so that errors due to the author's lack of knowledge of the technique of the mathematics involved may be eliminated. Certainly this should be done when an author thinks he has found a fallacy in a doctrine accepted as sound by mathematicians the world over.

# W. H. Bussey (1909)

Some Remarks on Mr. Russell's Article

### A. consensus

# B. the mathematical "we"

C. mathematical mysticism vs.philosophical perspective

It is very strange that Dr. Carus should consider there ought to be only one geometry, whereas we have various systems. We who inhabit the surface of one and the same planet have already constructed different geometries, and so why should there not be a possibility of the inhabitants of other heavenly bodies constructing other systems than one of those common among us? There may be beings who have attained a much higher degree of evolution than we; their mental faculties may transcend ours in an incredible degree of perfection.

#### Yoshio Mikami (1911)

Remarks on Dr. Carus's View Concerning Geometry

- A. consensus
- B. the mathematical "we"
- C. mathematical mysticism vs. philosophical perspective

The philosopher does not concern himself with the discovery of facts. He is content to get his facts ready made from the scientist. And it would perhaps be well if scientists would leave philosophy to philosophers. In science, the role of the philosopher is that of the spectator, who sees most of the game.

# W. E. Ayton Wilkinson (1907)

The Logical Aspect of the Theory of Hyper-spaces

- A. consensus
- B. the mathematical "we"
- C. mathematical mysticism vs. philosophical perspective

There have appeared in *The Monist*, from time to time, articles treating of "Hyper-space," "Non-Euclidean Geometry," "Pan-Geometry," or what might more appropriately be called "Pseudo-Geometry." I am the more surprised to see these articles go unchallenged in a periodical aiming so scrupulously at rigor in reasoning as does *The Monist*.

Charles H. Chase (1908) Pseudo-Geometry

- A. consensus
- B. the mathematical "we"
- C. mathematical mysticism vs. philosophical perspective

This communication on "Pseudo-Geometry" by Judge Charles H. Chase has a certain justification. It is an expression of common sense against being bulldozed into mysticism by the extravagances of a highly abstruse reasoning, and we endorse his protest so far as to say that we too do not believe in pseudo-geometry [...] which possesses little practical use because our old traditional geometry is simpler and better adapted to the solution of the problems of space-relations.

Paul Carus (1908) Editorial Comment

- A. consensus
- B. the mathematical "we"
- C. mathematical mysticism vs. philosophical perspective

These experts have wrapped up the doctrine in what to many is a maze of analytical language that requires a good deal of analytical erudition to compass and thoroughly possess. It makes the logician inclined to ask if these analysts have not mistaken some mere grammatical collocation of their analytical language for a real ideal possibility. We can say "round square," but nevertheless a "round square" is an absurdity. May not such analysts have made similar constructions?

Frances C. Russell (1909) A Modern Zeno

- A. consensus
- B. the mathematical "we"
- C. mathematical mysticism vs. philosophical perspective

And so it is but natural that these variations have excited the curiosity of only scholarly minds who have little or no interest in practical affairs and delight in the idea that there are vast regions of possibilities which have never been opened up to this commercially minded generation. It can scarcely be denied that the systems of Bolyai and Lobatchevsky command a purely theoretical interest, and so far as the pragmatic issues of life are concerned they may be regarded as stillborn children of the genius of mathematics.

# Paul Carus (1910)

Formal thought the basis of kenlore

- A. consensus
- B. the mathematical "we"
- C. mathematical

mysticism vs. philosophical perspective

I have proved how often genius is nervous epilepsy, and how almost all the sons of men of genius are lunatics, idiots, or criminals.

# Cesare Lombroso (1890)

The physiognomy of anarchists

- 1. prelude: "The Circle Squarer"
- 2. mathematics and *The Monist*
- 3. institutions and individuals
- 4. authority and dissent
- 5. words and meanings
- 6. agnotology and incommensurability

meaningful definitions and mathematical definitions

#### A. conservatism

- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

It is a prime requisite for a definition that the defining assertion assertions shall have a meaning, which is the same as to say that names must be employed that are already significant. These significant names must be so used that the intellectual sensibility shall be excited to perceive in a determinately that which is intended to be defined. In other words, sense and not nonsense must be produced in the mind that considers the definition.

# Edward Dixon (1892)

The Foundations of Geometry

#### A. conservatism

- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

So far as my linguistic feeling is concerned I cannot overcome the original meaning of the etymology of parallel. Parallel lines are to me lines which remain the same distance apart; thus railroad tracks, whether straight or curved, are parallel; and since this running side by side is the original sense of the word, they deserve to be so called. the geographer too uses the word parallel in its etymological meaning. If straight lines are parallel (i.e., keep at the same distance) they do not meet even in infinity, —although I will grant anything for infinity.

# Paul Carus (1910)

The Nature of Logical and Mathematical Thought

# A. conservatism

# B. parallels

- C. space
- D. straight lines
- E. mathematical definitions

The real meaning of the statement is that, when we consider the distance of parallels apart, it may be neglected as insignificant in comparison, when treating of lines infinitely long. As a matter of real truth the parallels are as far apart at the infinite distance as they were at the place of beginning. That is, indeed, true by our definition of parallels.

Charles H. Chase (1908)

Pseudo-Geometry

A. conservatism

# B. parallels

- C. space
- D. straight lines
- E. mathematical definitions

It seems me that Dr. Halsted, + the rest of our rabid "meta geometricians" can find no fault with this definition of parallel lines; which is my own.

Defining a line as direction. Then
(a) Parallel right lines are those which lie at an invariable distance from each other. And thus have the same direction.
(b) Hence, at whatever distance they may be produced, even though it be at infinity, they can never approach it, or retreat from each other in the same direction.

slightest degree. Still less, then, can they ever cut.

Alan Hawkesworth (1909) Letter to Paul Carus

A. conservatism

# B. parallels

- C. space
- D. straight lines
- E. mathematical definitions

A parallel to a straight meets it at infinity. An ultraparallel does not even meet it at infinity. Parallels are straights with a common point at infinity. Parallels are straights which meet on a figurative point. If ultraparallels determine a point, we must have another name for it. Call it an ideal point. Thus equipped, we are able to answer Mr. Russells' question, Monist, page 621: "How is the professional expert (the man who knows non-Euclidean geometry) better fitted to see more lucidly in dealing with the elements of geometry than any other person of good geometric faculty?"

# George Bruce Halsted (1910)

Bolyai, Lobatchevsky, Russell

- A. conservatism
- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

Larger meanings must have time to grow; the smaller ones, those that are most natural and most provincial, being also the most persistent. In the process of clarification, expansion and readjustment, his fine old word, space, early come into his life and gradually stained through and through with the refracted partial lights and multi-colored prejudices of his youth, is not to be robbed of its proper charms nor to be shorn of its proper significance.

**Cassius Keyser (1906)** Mathematical Emancipations

- A. conservatism
- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

Volumes have been written by psychologists and philosophers to explain the notion of space; and the more these philosophers write, the more they seem to think there is something mysterious about it, and the more they themselves become befogged in their reasoning about it. It is a mere matter of definition; everyone knows what it is and can usually define what he means by the word. One might as well attempt to prove that a straight line is straight, or that a circle is circular.

Charles H. Chase (1908)

Pseudo-Geometry

- A. conservatism
- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

What is a point except a spot in space? Professor Russell must excuse me for finding Professor Fano's method of avoiding the difficulty comical. [...] This is all very nice and begins like a fairy-tale, "Once upon a time." He rushes these statements upon us with an unmitigated abruptness which is truly naive. He has points, lines, distances, directions, but knows nothing of space. The very word "space" is abolished! Such are "the rigid methods employed by modern geometers" that "have deposed Euclid form his pinnacle of correctness"!

# Paul Carus (1910)

The Nature of Logical and Mathematical Thought

- A. conservatism
- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

Dr. Carus speaks always as though non-Euclidean straight lines were not really straight, but were merely called straight out of willfulness. The projective treatment shows, better than the metrical, wherein the straight lines of non-Euclidean spaces agree with those of Euclid, and ought therefore not to be omitted even in a mere outline.

**Bertrand Russell (1910)** REVIEWS: The Foundations of Mathematics (Carus) *Mathematical Gazette* 

- A. conservatism
- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

What is needed is a definition of the straight line is a statement or conspiracy of statements that shall express and exhibit to the intelligence the matters of fact in virtue of which it has that quality we call straightness. If this can be done in any other way than by defining a rule according to which the points that stud it are distributed so as to make it straight, then I am at a loss to conceive what that way can be. Moreover the straight line ought to be defined so as to put right out in space in perfect self-sufficiency.

Francis C. Russell (1909)

A Modern Zeno

- A. conservatism
- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

Not only has mathematical logic led to a recognition of the fact that to speak of "definability" or "nondefinability" of a given word or concept is to use a meaningless phrase, so long as no precise indication is given as to what other words or concepts may be used in the desired definition; but it has also afforded an explanation of the fact that many among the most important words in science and philosophy are found to be among those very ones of which it is unreasonable to ask or to seek a definition, in the scholastic sense.

# Giovanni Vailati (1906)

Pragmatism and Mathematical Logic

- A. conservatism
- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

In considering any attempt to prove Euclid's parallel postulate, it is well to consider first what is meant by geometrical proof. In defining some terms by means of others, it is obvious that we must begin primarily with some terms which are themselves left undefined.

G. W. Greenwood (1909)

A Twentieth Century Zeno

- A. conservatism
- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

A well-chosen word usually suffices to do away with the exceptions from which the rules stated in the old way suffer; this is why we have created negative quantities, imaginaries, points at infinity, and what not. And exceptions, we must not forget, are pernicious because they hide the laws.

Henri Poincaré (1910) Future of Mathematics

- A. conservatism
- B. parallels
- C. space
- D. straight lines
- E. mathematical definitions

I regret that the valuable consequences I derived from his formulas were gross errors into which I was led by assuming that a man that undertook to give mathematical demonstrations of the errors of great mathematicians was using the language and methods of mathematics.

Antonio Llano (1900) More Strange Mathematics and Mechanics

- 1. prelude: "The Circle Squarer"
- 2. mathematics and *The Monist*
- 3. institutions and individuals
- 4. authority and dissent
- 5. words and meanings
- 6. agnotology and incommensurability

some conclusions about mathematical practices
# A. the same Beoetians?

- B. mere ignorance
- C. American ignorance
- D. incommensurable
- E. an American revolution?

Most of them dismissed non-Euclidean geometry as a logically viable but physically meaningless intellectual exercise, and sought the unshakeable foundation of established geometry in a geometrical intuition which they believed was common to all mankind. Apriorists were not alone in their rejection of the physical and, consequently, the philosophical significance of the new geometries, but were joined by some empiricists, who staunchly defended the exclusive validity of Euclidean geometry.

# Roberto Torretti (1978)

Philosophy of Geometry from Riemann to Poincaré

### A. the same Boetians?

- B. mere ignorance
- C. American ignorance
- D. incommensurable
- E. an American revolution?

The uproar Gauss had feared came after his death. The strongest protests were made by philosophers who would not admit any tampering with Euclidean geometry, the established paradigm of scientific knowledge. Many of the objections raised against the novel geometric conceptions merely showed ignorance (and a remarkable readiness to believe mathematicians guilty of the wildest nonsense).

# Roberto Torretti (1978) Philosophy of Geometry from Riemann to Poincaré

A. the same Boetians?

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- C. American ignorance
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The uproar Gauss had feared came after his death. The strongest protests were made by philosophers who would not admit any tampering with Euclidean geometry, the established paradigm of scientific knowledge. Many of the objections raised against the novel geometric conceptions merely showed ignorance (and a remarkable readiness to believe mathematicians guilty of the wildest nonsense).

# Roberto Torretti (1978) Philosophy of Geometry from Riemann to Poincaré

A. the same Boetians?

### B. mere ignorance

- C. American ignorance
- D. incommensurable
- E. an American revolution?

That not every one can invent is nowise mysterious. That not every one can retain a demonstration once learned may also pass. But that not every one can understand mathematical reasoning when explained appears very surprising when we think of it. And yet those who can follow this reasoning only with difficulty are in the majority: that is undeniable, and will surely not be gainsaid by the experience of secondary school teachers.

#### Henri Poincaré (1910)

Mathematical creation trans. George Bruce Halsted

# A. the same Boetians?

### B. mere ignorance

- C. American ignorance
- D. incommensurable
- E. an American revolution?

Ignorance, far from being a simple lack of knowledge that good science aims to banish, is better understood as a practice with supporting social causes as complex as those involved in knowledge practices.

# Nancy Tuana (2008)

"Coming to Understand" Agnotology: The Making and Unmaking of Ignorance

- A. the same Boetians?
- B. mere ignorance
- C. American ignorance
- D. incommensurable
- E. an American revolution?

As Robert Proctor argued in his study of the politics of cancer research and dissemination, *Cancer Wars* (1995), we must "study the social construction of ignorance. The persistence of controversy is often not a natural consequence of imperfect knowledge but a political consequence of conflicting interests and structural apathies. Controversy can be engineered: ignorance and uncertainty can be manufactured, maintained, and disseminated."

#### Nancy Tuana (2008)

"Coming to Understand" Agnotology: The Making and Unmaking of Ignorance

- A. the same Boetians?
- B. mere ignorance
- C. American ignorance
- D. incommensurable
- E. an American revolution?

Agnotology traces the cultural politics of ignorance. It takes the measure of our ignorance, and analyzes why some knowledges are suppressed, lost, ignored, or abandoned, while others are embraced and come to shape our lives. Ignorance is often not merely the absence of knowledge but an outcome of cultural struggles.

# Londa Schiebinger (2008)

"West Indian Abortifacients and the Making of Ignorance"

Agnotology: The Making and Unmaking of Ignorance

- A. the same Boetians?
- B. mere ignorance
- C. American ignorance
- D. incommensurable
- E. an American revolution?

As to the influence which the intellect of one man may have on that of another, it must necessarily be very limited in a country where the citizens, placed on an equal footing, are all closely seen by each other; and where, as no signs of incontestable greatness or superiority are perceived in any one of them, they are constantly brought back to their own reason as the most obvious and proximate source of truth.

# Alexis de Tocqueville (1840)

Influence of Democracy upon the action of intellect in the United States

- A. the same Boetians?
- B. mere ignorance
- C. American ignorance
- D. incommensurable
- E. an American revolution?

As they perceive that they succeed in resolving without assistance all the little difficulties which their practical life presents, they readily conclude that everything in the world may be explained, and that nothing in it transcends the limits of the understanding. Thus they fall to denying what they cannot comprehend; which leaves them but little faith for whatever is extraordinary, and an almost insurmountable distaste for whatever is supernatural.

# Alexis de Tocqueville (1840)

Influence of Democracy upon the action of intellect in the United States

- A. the same Boetians?
- B. mere ignorance
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There is a cult of ignorance in the United States, and there always has been. The strain of antiintellectualism has been a constant thread winding its way through our political and cultural life, nurtured by the false notion that democracy means that "my ignorance is just as good as your knowledge."

Isaac Asimov (1980)

"A Cult of Ignorance" Newsweek

- A. the same Boetians?
- B. mere ignorance
- C. American ignorance
- D. incommensurable
- E. an American revolution?

[...] the editor is seriously requested to submit manuscripts to a mathematician (presumably an orthodox non-Euclidean) and to suppress all heretical articles. In reply to this request I will state that I frequently publish articles setting forth views which I do not endorse, because I believe that they are worth being noticed, considered and perhaps refuted.

**Paul Carus (1909)** Construction of the Straight Line

- A. the same Boetians?
- B. mere ignorance

# C. American ignorance

- D. incommensurable
- E. an American revolution?

Why did some contributors promote ignorance of modern mathematics?

To what extent did this show of ignorance reflect contemporary changes in mathematical authority?

What were the impacts for mathematical practices?

- A. the same Boetians?
- B. mere ignorance
- C. American ignorance

### D. incommensurable

E. an American revolution?

What is often forgotten is that the problem with which geometers had been coping for centuries was whether what the remaining axioms of Euclidean geometry *say* already contains the information conveyed by the fifth postulate, and *not* whether the postulate is contained in the remaining axioms when you ignore the meanings of all geometric words.

Alberto Coffa (1983) From Geometry to Tolerance

- A. the same Boetians?
- B. mere ignorance
- C. American ignorance

#### D. incommensurable

E. an American revolution?

Broadly speaking to say that a pair of theories is incommensurable is to say that the theories do not share a common language, or that the terms they employ do not have common meaning.

The languages of competing or successive theories in the same domain may differ with respect to the meaning, and even the reference, of their terms.

Howard Sankey (1994)

The Incommensurability Thesis

- A. the same Boetians?
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- C. American ignorance

### D. incommensurable

E. an American revolution?

Within the new paradigm, old terms, concepts, and experiments fall into new relationships one with the other. The inevitable result is [...] a misunderstanding between the two competing schools [...] Communication across the revolutionary divide is inevitably partial.

**Thomas Kuhn (1970)** The Structure of Scientific Revolutions (2nd ed.)

- A. the same Boetians?
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This suggests that we may distinguish two types of revolution. In the first type, which could be called *Russian*, [...] 'some previously existing entity...is overthrown and irrevocably discarded'. In the second type, which could be called *Franco-British* the 'previously existing entity' persists, but experiences a considerable loss of importance.

# Donald Gillies (1991)

Introduction to Revolutions in Mathematics

- A. the same Boetians?
- B. mere ignorance
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In mathematics, revolutions do occur but they are always of Franco-British type. An innovation in mathematics (or a branch of mathematics) may be said to be a revolution if two conditions are satisfied. First of all, the innovation should change mathematics (or the branch of mathematics) in a profound and farreaching way. Secondly, the relevant older parts of mathematics, while persisting, should undergo a considerable loss of importance.

# Donald Gillies (1991)

Introduction to Revolutions in Mathematics

- A. the same Boetians?
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[...] the discovery of non-Euclidean geometries transformed our conception of the nature of space and of geometry itself, profoundly changed our understanding of geometric 'objects', and introduced new 'entities', concepts, and methods, but it did not prove that Euclidean geometry was false or useless it simply showed that the Euclidean viewpoint was not a unique, universal mathematical theory and that, as an abstract theory claiming to describe physical events, it has only limited and relative validity.

# Luciano Boi (1991)

The 'revolution' in the geometrical vision of space in the nineteenth century, and the hermeneutical epistemology of mathematics

- A. the same Boetians?
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- C. American ignorance
- D. incommensurable
- E. an American revolution?

I believe that when the present fad of the non-Euclideans has died out my time will come. I am confident of final success, not in the present generation but in the coming one. I am in no hurry to press my claim. In the meantime I shall be glad to let our readers become acquainted with Enriques' mathematical ideas.

### Paul Carus (1909)

Letter to Josiah Royce re: translation of Enriques' Problems of Science

- A. the same Boetians?
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- C. American ignorance
- D. incommensurable
- E. an American revolution?

The success of non-euclidean geometry resulted in a secession of elite from majority mathematics practitioners.

To what extent should both sides inform studies of mathematical practices?

