Certain Modern Ideas and Methods

Charlotte Angas Scott's Philosophy of Mathematics

Association for Women in Mathematics,
Special Session in the History of Mathematics
Jemma Lorenat, Pitzer College
jlorenat@pitzer.edu

"I am expecting to meet Mr. Russell today (I am spending a few days in Cambridge), to talk over the lectures he is to give us. He seems to be getting quite a reputation in Cambridge among those whose opinion is worth having, and I feel very glad that we are to have this opportunity of hearing him."

Charlotte Angas Scott (1896)
Letter to Carey Thomas

"The communications made in Sections V and VI, while not necessarily the most valuable mathematically, were yet of the most general interest, and lend themselves best to any general report. The sitting was opened by M. Hilbert's address, in German, on the future problems of mathematics."

Charlotte Angas Scott (1900)
"The International Congress of Mathematicians in Paris"
Bulletin of the American Mathematical Society

Some Principles of Physical Science

A LECTURE BY

PROFESSOR ALFRED NORTH WHITEHEAD, Sc.D., F.R.S.

PROFESSOR OF APPLIED MATHEMATICS IN THE
IMPERIAL COLLEGE OF SCIENCE,
SOUTH KENSINGTON

DEDICATED TO

PROFESSOR CHARLOTTE ANGAS SCOTT, D.Sc.

HEAD OF THE DEPARTMENT OF MATHEMATICS
IN BRYN MAWR COLLEGE

AND READ BEFORE A MEETING HELD IN HER HONOR
BY HER FORMER STUDENTS

"The strict limitation to a time previously determined would in most cases be beneficial to the author, obliging him to select, subordinate, and group his details. It is tolerably certain that if the author regards all details as equally important, his auditors will regard all as equally unimportant."

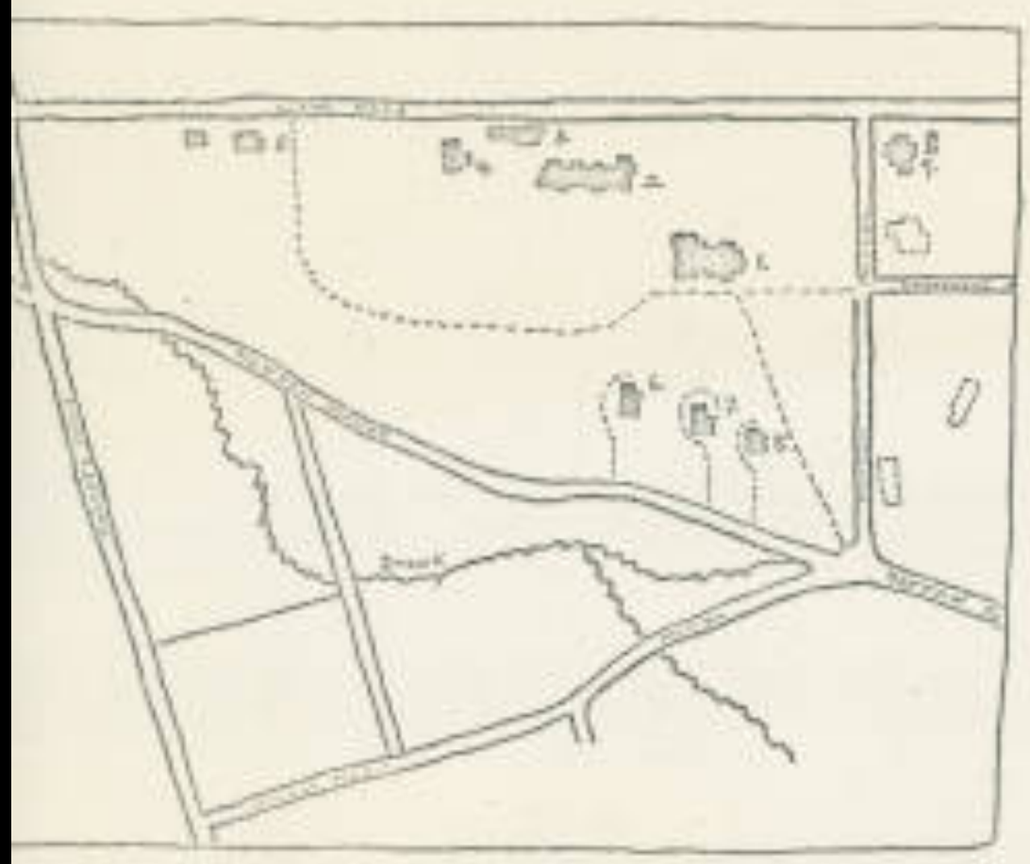
Charlotte Angas Scott (1900)
"The International Congress of Mathematicians in Paris"
Bulletin of the American Mathematical Society

1. Moral philosophy: standards of mathematics education for women

2. Ontology: the relationship between algebra and geometry

3. Conclusions: who was a philosopher of mathematics?

1885



1885

Bryn Mawr College

1. Taylor Hall
2. Merion Hall
3. Physics Laboratory
4. Gymnasium
5. Old Farm House
6. The Deanery
7. Yarrow [Betweenery]
8. Keiserhof [Greenery]
9. Cartres

"Professor Harkness believes Scott's distinction at Cambridge marks the turning point in England from the theoretical feminism of Mill and others to the practical educational and political advances of the present time."

Emily James Putnam (1922)
"Celebration in Honor of Professor Scott"
Bryn Mawr Alumnae Bulletins

"I think it will be best for Mr. Harkness *not* to be present then, as I shall be obliged to ask you to make clear to me what your view of the possibilities of education for women really is. I am most disturbed and disappointed at present to find you taking the position that intellectual pursuits must be "watered down" to make them suitable for women, and that a lower standard must be adopted in a woman's college than in a man's."

Charlotte Angas Scott (1898)
Letter to Carey Thomas

"I do not expect any of the other members of the faculty to feel in this way about it; they, like nearly all other men that I have known, doubtless take an attitude of toleration, half-amused and half kindly, on the whole question; for even where men are willing to help in women's education, it is with an inward reserve of condescension."

Charlotte Angas Scott (1898)
Letter to Carey Thomas

Miss Caroline N. Martin.
Mount Holyoke prof. of Math.

"The opponents of the subject in a woman's college, especially if they are themselves women, are somewhat wary of asking that time-honored question. They know that they are supposed to claim the same stiff intellectual training for women as for men, so they avoid stating the question in bald terms of sex. Instead they say: "Every student who enters our college has already had several years of mathematics. She has enough for all practical purposes.""

Emilie Martin (1917)
"Relating to Required Mathematics for Women Students"
The American Mathematical Monthly

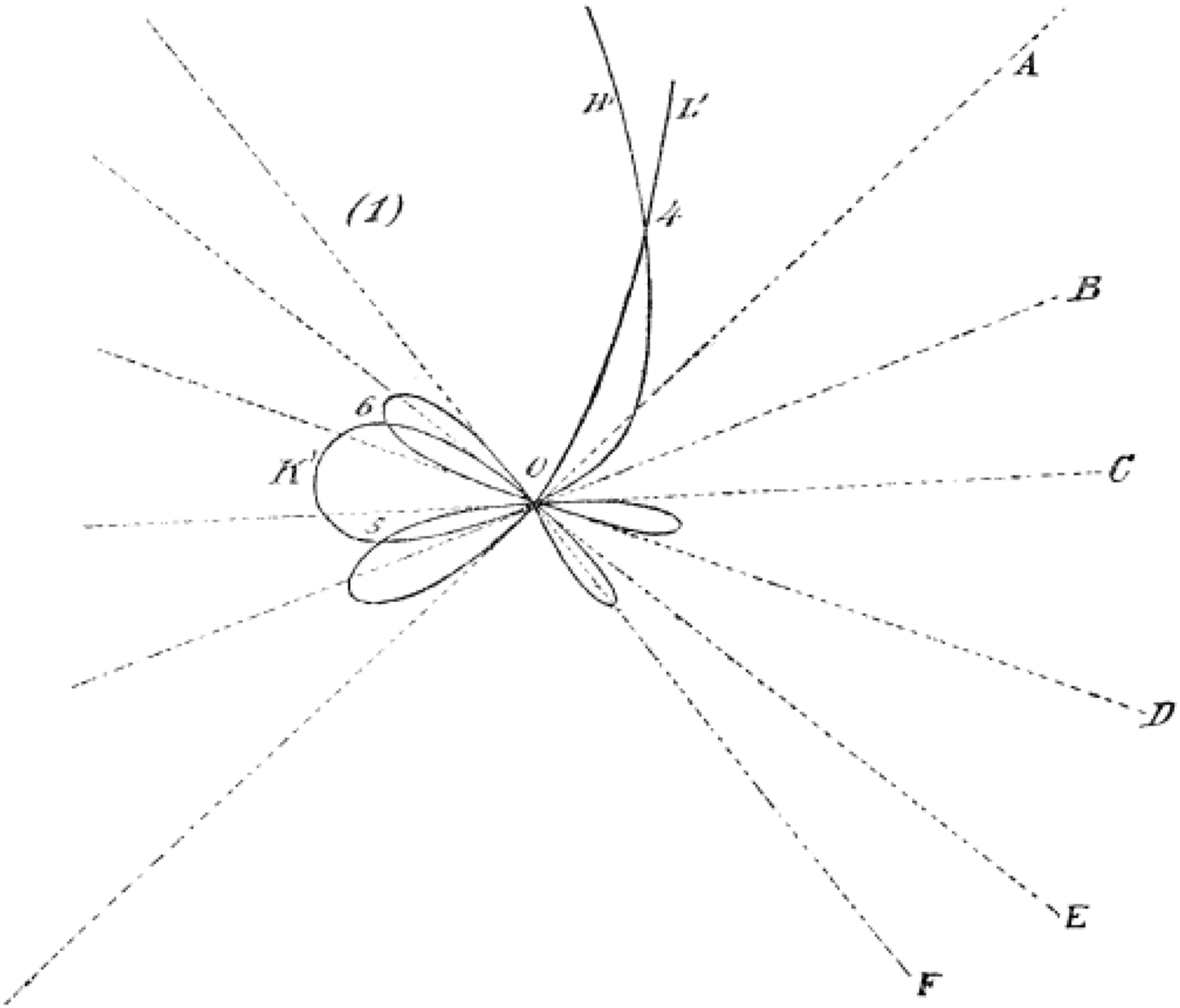
1. Moral philosophy: standards of mathematics education for women
    - opportunities for advanced mathematics research
    - commitment to uncompromising undergraduate curriculum
2. Ontology: the relationship between algebra and geometry
3. Conclusions: who was a philosopher of mathematics?

"Is there any geometrical reality corresponding to the algebraic symbol? Do the numbers *d*, *x* express actual facts, or are they conventional representation of the point to satisfy Plücker's equations?"

Scott (1892)
"On the Higher Singularities of Plane Curves"
American Journal of Mathematics

"But an algebraic curve has a geometric existence, and the process here used for the analysis of singularities is a geometric process depending entirely on a geometric conception of singularities, even though the language used is necessarily algebraic."

Scott (1893)
"The Nature and Effect of Singularities in Plane Algebraic Curves"
American Journal of Mathematics



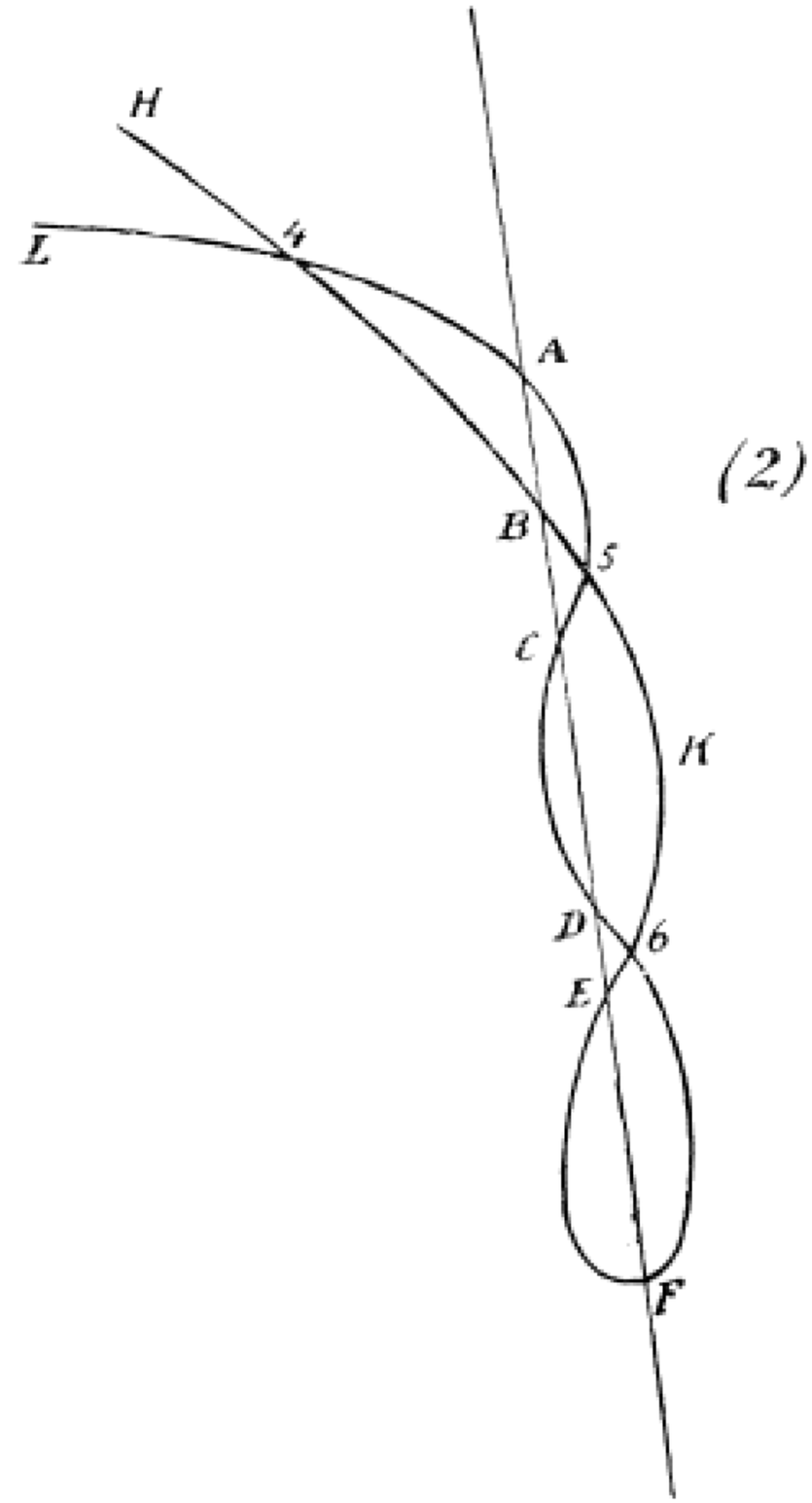
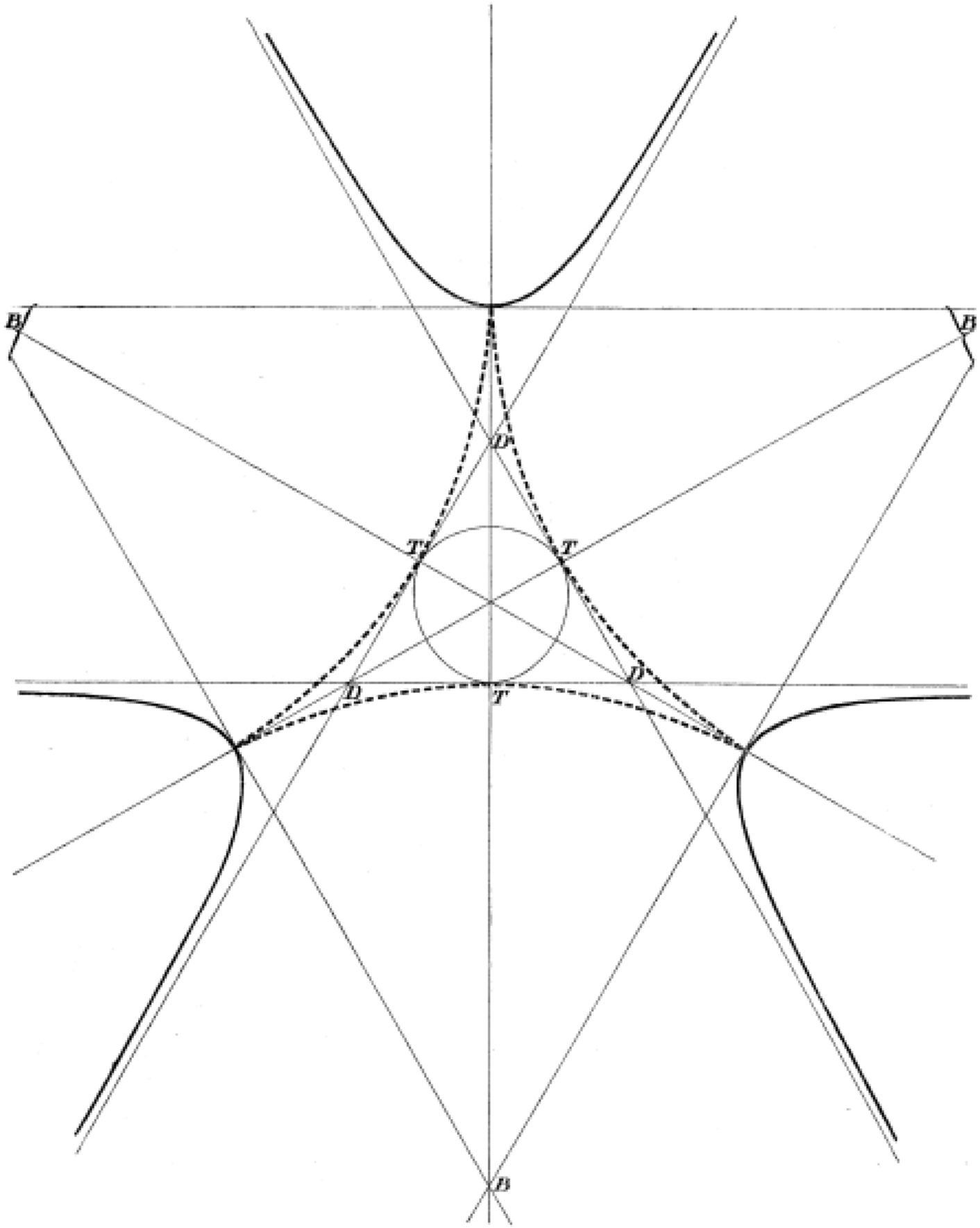


FIG. 26.

"In the case of a singularity with coincident tangents, the danger in separating the tangents is that the neighboring multiple points may be destroyed. But these are indicated by multiple points on the base line; we can therefore separate the tangents provided that, on the inverse, no multiple points are thereby lost in the immediate neighborhood of the base line; and we can substitute nodes and loops for the cusps, by displacing the base line so that there may no longer be contact."

Scott (1892)
"On the Higher Singularities of Plane Curves"
American Journal of Mathematics

Fig. 12.



"The diagrams here given have been made by means of 12; but from the analytical expressions just quoted a graph can be constructed, by means of which these may be readily drawn, and the variation possibly more easily grasped."

Scott (1894)
"On Plane Cubics"
Philosophical Transactions of the Royal Society of London

Fig. 32.

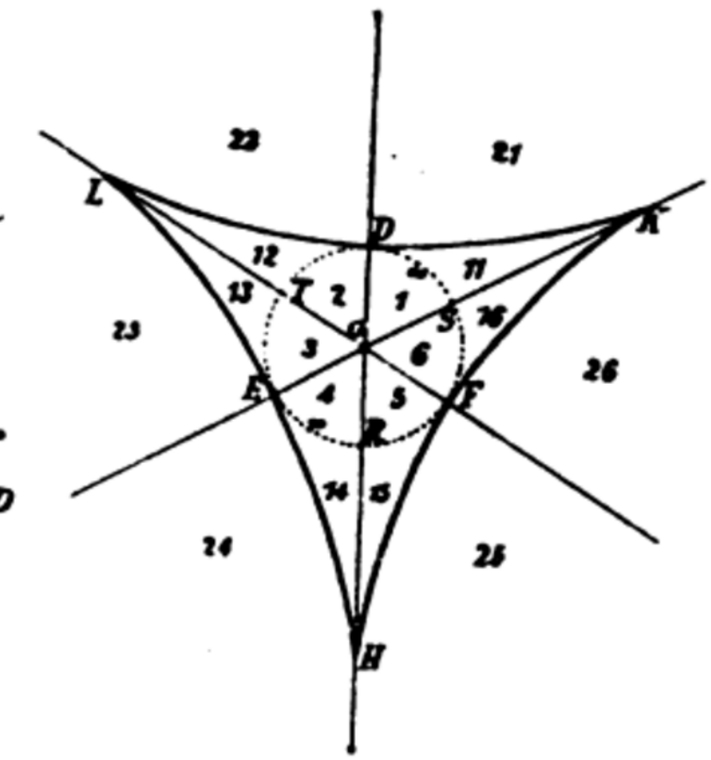


Fig. 36.

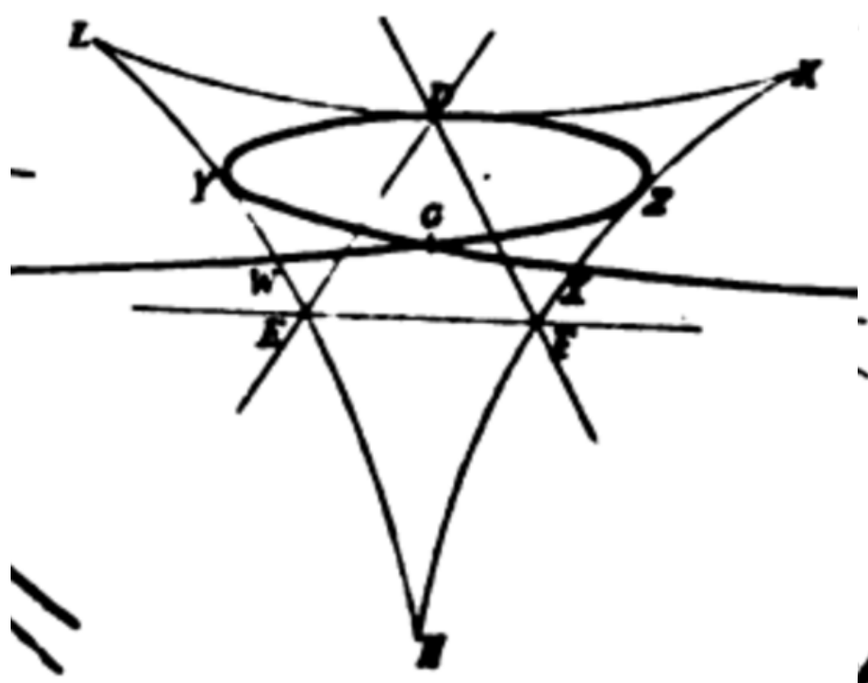
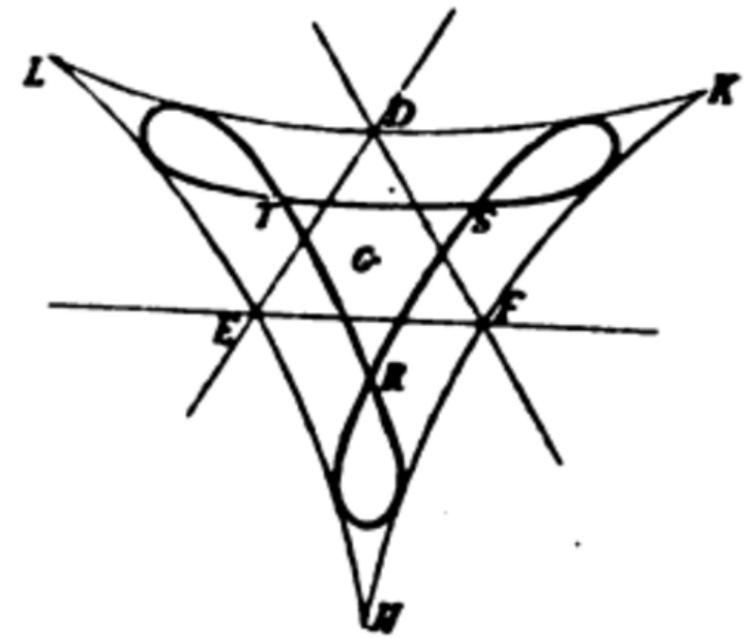


Fig. 33.



"By means of these compartments the effect of transformation on any curve is made graphically intuitive; and they, or rather the curves by which they are obtained, are utilised in the more formal algebraic proofs […] the compartments themselves can be used in the proofs without impairing the rigour of the argument. The compartment idea is intended to appeal, not to naïve intuition, but to refined intuition."

Scott (1899)
"Studies in the transformation of plane algebraic curves"
The Quarterly Journal of Pure and Applied Mathematics

"Professor Klein, in his Evanston Lectures, dividing mathematicians into logicians, formalists, and intuitionists, classes him as a formalist, a term not altogether pleasing, inasmuch as it conveys a shade of reproach, but which is immediately explained by the speaker. [...] It has always seemed to me that while Cayley's processes were algebraic, since the language of algebra was simpler to him than the ordinary language of words, the color of his thought was essentially geometrical."

Scott (1895)
"Arthur Cayley"
Bulletin of the American Mathematical Society

"In concluding, I wish to insist in particular on what I regard as the principal characteristic of the geometrical methods that I have discussed to-day: these methods give us an *actual mental image* of the configuration under discussion, and this I consider as most essential in all true geometry."

Felix Klein (1894)
The Evanston Colloquium : Lectures on Mathematics

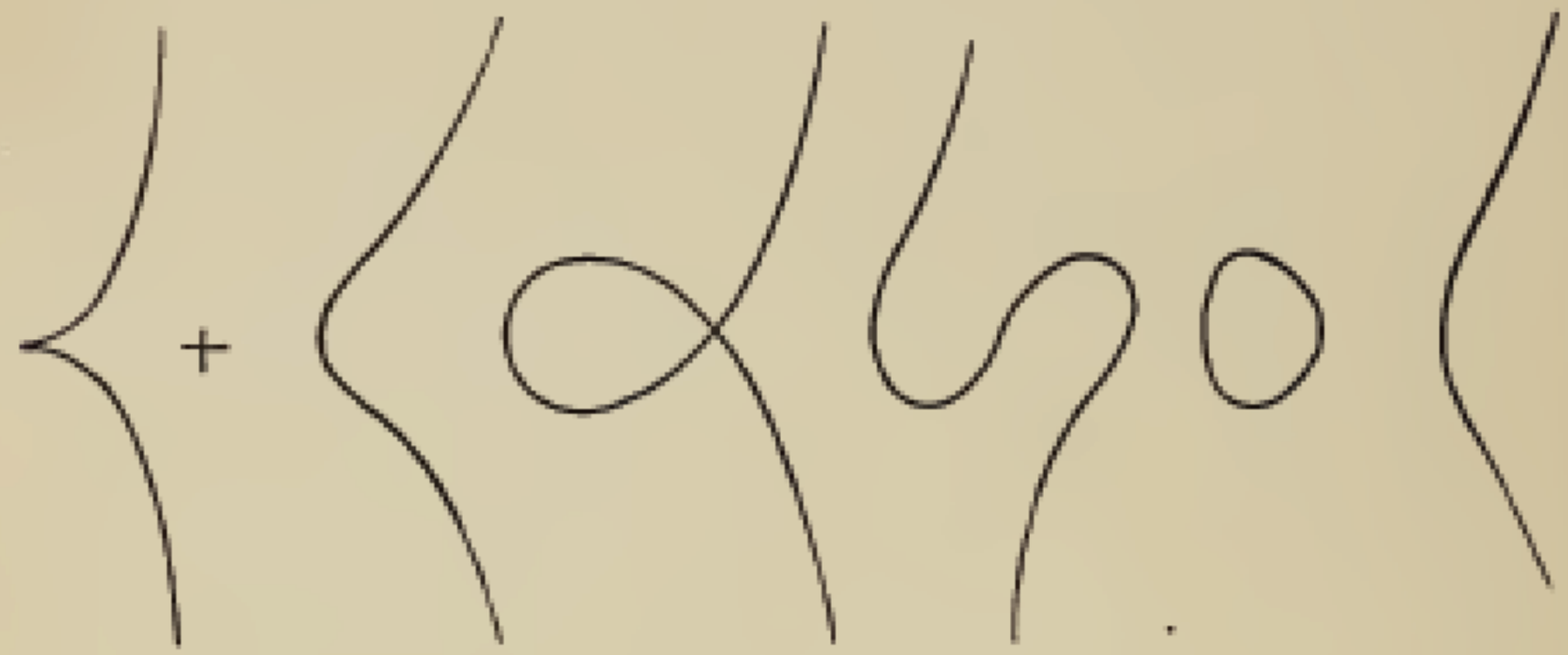


Fig. 1.

Fig. 2.

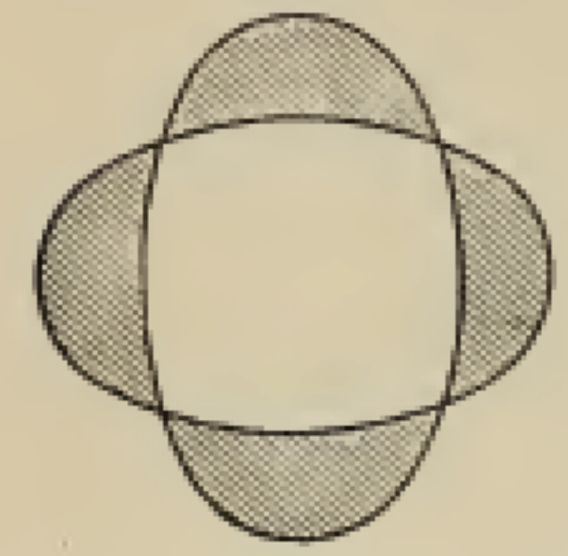


Fig. 3.

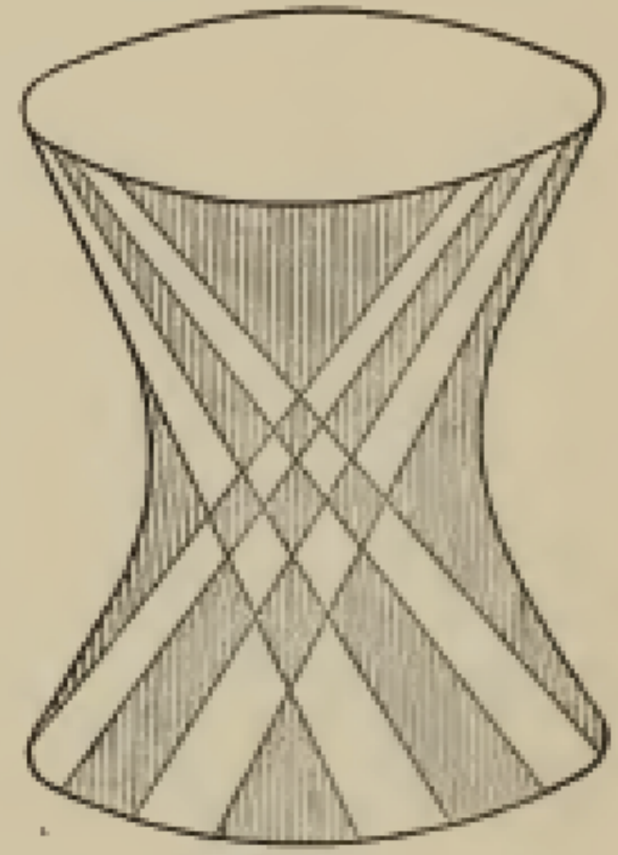


Fig. 4.

"If we now ask how we can account for this distinction between naïve and refined intuition, I must say that, in my opinion, the root of the matter lies in the fact that *the naïve intuition is not exact, while the refined intuition is not properly intuition at all, but arises through the logical development from axioms considered as perfectly exact."*

Felix Klein (1894)
The Evanston Colloquium : Lectures on Mathematics

AN INTRODUCTORY ACCOUNT

OF

CERTAIN MODERN IDEAS AND METHODS

IN

PLANE ANALYTICAL GEOMETRY

BY

CHARLOTTE ANGAS SCOTT, D.Sc.

GIRTON COLLEGE, CAMBRIDGE; PROFESSOR OF MATHEMATICS IN BRYN MAWR COLLEGE, PENNSYLVANIA

"In analytical geometry the subject-matter is geometry while the language is algebraic. For progress and pleasure it is of primary importance that the language be properly adjusted to the subject; elasticity must be preserved and unnecessary restrictions cast aside."

Scott (1894)
Certain Modern Ideas and Methods in Plane Analytical Geometry

40. It has been shown that any line through the intersection of $u=0$, $v=0$ has an equation of the form $u-kv=0$; and that the value of u at any point P is some multiple of the perpendicular from P to the line $u=0$, that is, $u=l\alpha$, and similarly $v=m\beta$. Writing p for $u-kv$, Fig. 14 shows that

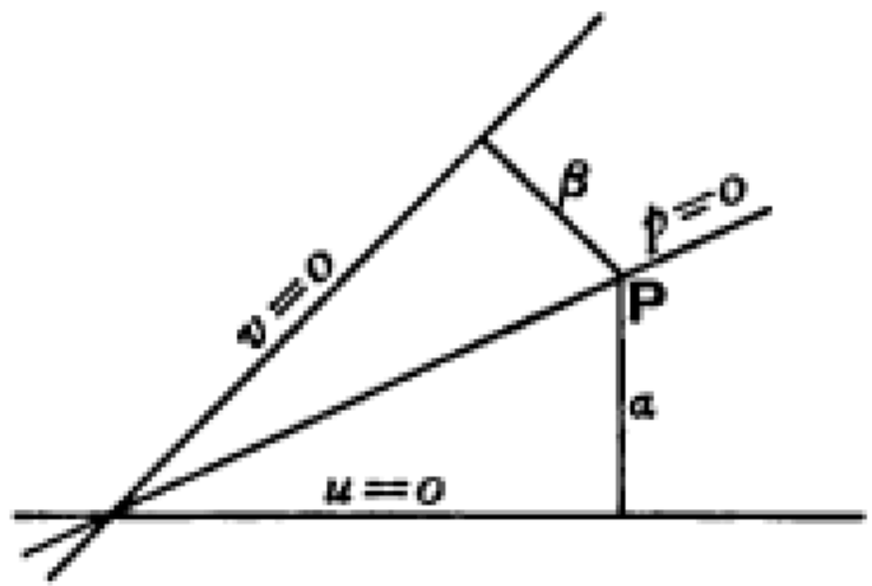


FIG. 14.

$$\frac{\sin up}{\sin pv}=\frac{\alpha}{\beta}$$
$$=\frac{m}{l}\cdot\frac{u}{v}$$
$$=\frac{m}{l}k,$$

since at P, $u-kv=0$.

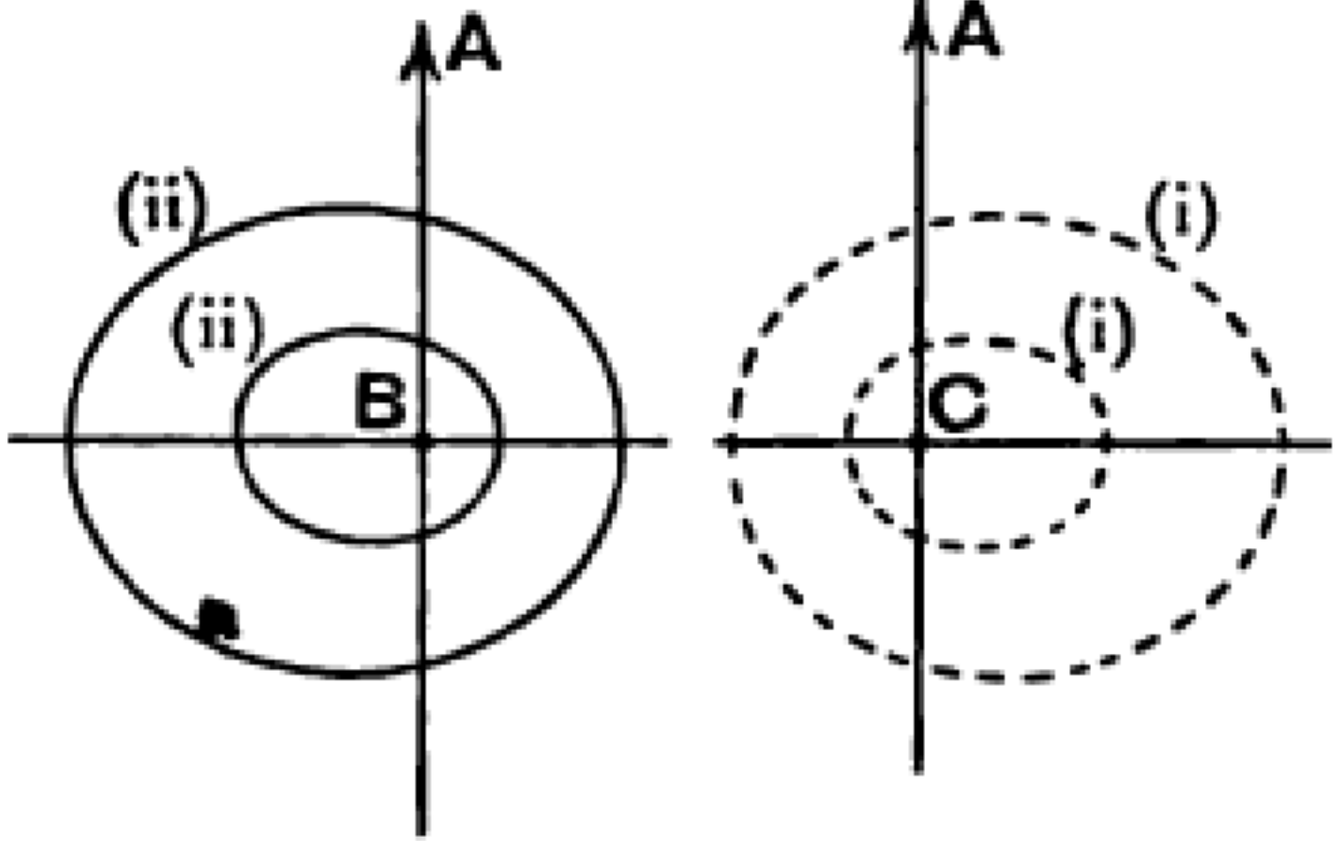
Similarly taking another line $q=u-k'v=0$,

$$\frac{\sin uq}{\sin qv}=\frac{m}{l}k'.$$

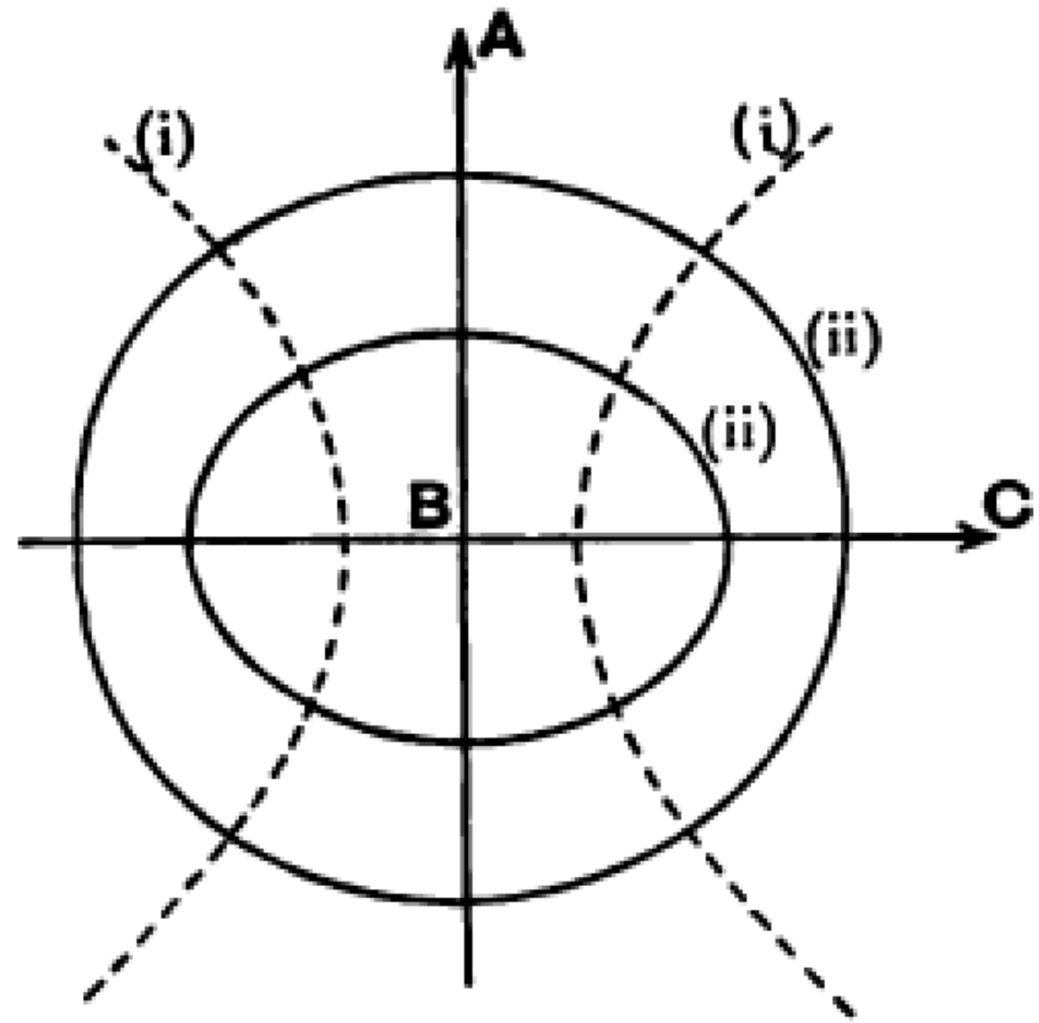
"Imaginaries present themselves naturally in the solution of algebraic equations, and are then recognized for the sake of continuity. If now we refuse to admit them into algebraic geometry, we shall have to examine the work at every step, to see whether it has a legitimate application in geometry; our symbolical language will no longer have an exact relation to the subject matter.

Note. The introduction of imaginary elements into Pure Geometry depends on different considerations, and requires independent justification. "

Scott (1894)
Certain Modern Ideas and Methods in Plane Analytical Geometry



(b)



(b)

"Though these exceptional line elements belonging to the system cannot be directly represented in the diagram, being imaginary, yet they can be exactly indicated by means of their real intersections. Similarly the exceptional point elements of a curve cannot be marked on the diagram, for being at infinity they are beyond the limits; but they can be indicated by means of the asymptotes."

Scott (1894)
Certain Modern Ideas and Methods in Plane Analytical Geometry

THE

MATHEMATICAL GAZETTE.

EDITED BY

W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF

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E. T. WHITTAKER, M.A.

LONDON:

GEORGE BELL & SONS, YORK ST., COVENT GARDEN,

AND BOMBAY.

ON VON STAUDT'S GEOMETRIE DER LAGE.

It is far too much the custom now to rely on the analogy of algebra to justify the introduction of imaginaries into geometry. Analogy, however, is no justification unless we first prove the exact correspondence of the fields of investigation. In analytical geometry the identification of the two fields is permissible, and is easily explained; but in pure geometry any reference to algebra, expressed or implied, is irrelevant and misleading. The elements of pure geometry have no dependence on calculation.

"It is one of the axioms of modern mathematics that von Staudt placed the doctrine of imaginaries on a firm geometrical basis ; but logical and convincing as his treatment is, when patiently studied in all its detail, it yet seems to me hardly practicable as a class-room method. [...] the apparent break in passing from the seen to the unseen, which arouses scepticism as to whether the formal elements can truly be said to "exist." While the reason, if sufficiently trained, is convinced, all natural instincts rebel. The whole thing impresses the natural man as simply a tour de force. "

Scott (1899)
"The Status of Imaginaries in Pure Geometry"
Bulletin of the American Mathematical Society

"This is one of the best surveys of von Staudt's thought and, unlike the notoriously turgid original, it attempts to articulate the philosophy behind the method."

Mark Wilson (1992)
"Frege: The Royal Road from Geometry"
Frege's Philosophy of Mathematics

1. Moral philosophy: standards of mathematics education for women
2. Ontology: the relationship between algebra and geometry
    - geometrical existence and mental images
    - naïve versus refined intuition
    - natural instincts versus logical reasoning
3. Conclusions: who was a philosopher of mathematics?

"Unfortunately, I am not in a position to give a full account of the opinions of philosophers on this subject."

Felix Klein (1894)
The Evanston Colloquium : Lectures on Mathematics

“We believe that our reasoning no longer appeals to intuition; the philosophers tell us that this is an illusion.”

Henri Poincaré (1900)
“The Role of Intuition and Logic in Mathematics”

"So far as I can make out, Dr. Schmidt has not specialized in Pure Mathematics; he has diverged on the one hand into Physics, on the other into Philosophy. This last appears to be his principal interest, and I should suppose that his lectures on the Foundations of Mathematics have been quite as much on the side of Philosophy as on Mathematics. I am very dubious about his qualifications for even a temporary appointment, unless supplemented by advanced courses from outsiders."

Scott (1903)
Letter to M. Carey Thomas

1. Moral philosophy: standards of mathematics education for women
2. Ontology: the relationship between algebra and geometry
3. Conclusions: who was a philosopher of mathematics?
   - status of philosophy among mathematicians
   - gendered categories: philosophical or pedagogical
   - contributions to current and future areas of inquiry

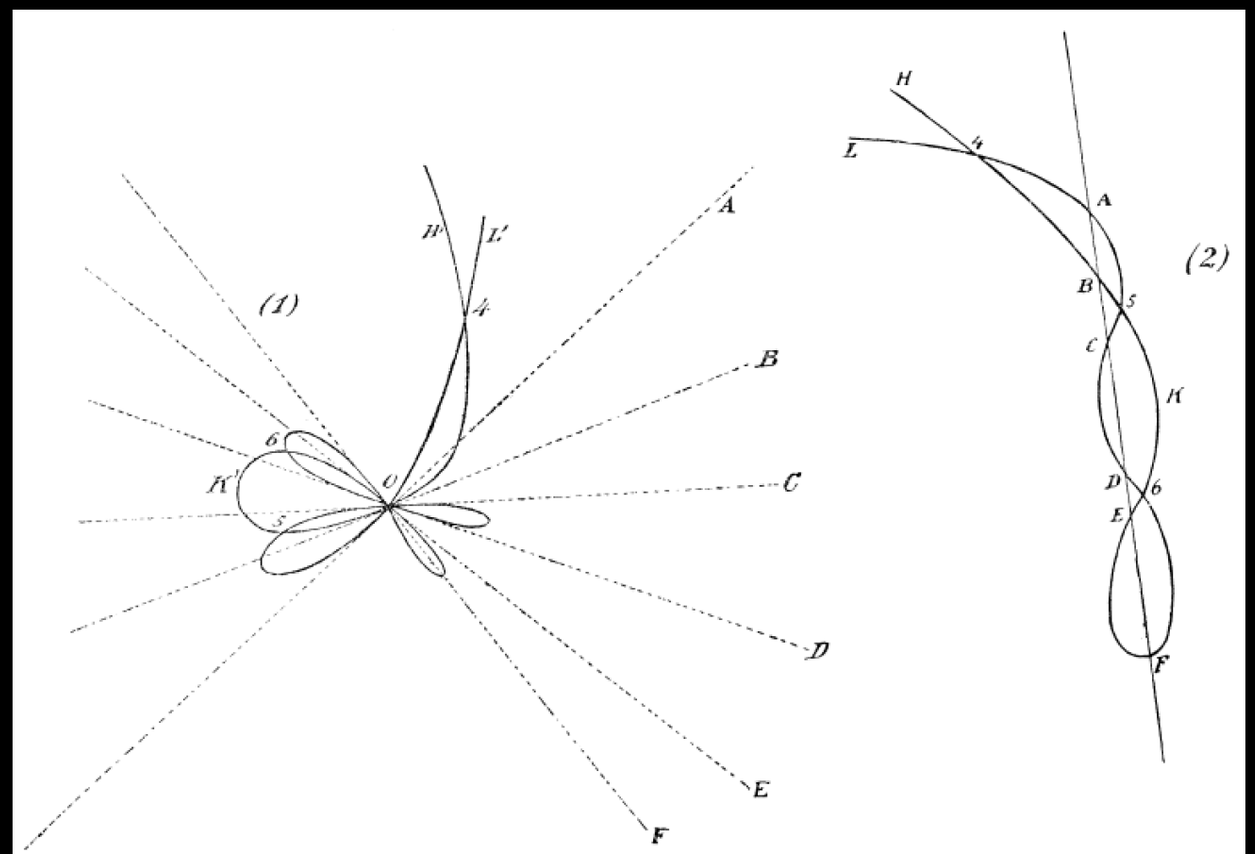

Fig. 26.

Thank you.

"Note. It was shown that if from an equation F=0 we derive a new equation F'=0 by a linear transformation, then (i.) the two curves F=0 and F'=0 can be placed in perspective; (ii.) the two equations F=0 and F'=0 can be regarded as two equations of the same curve with different triangles of reference.

The former is the natural view when the subject is considered geometrically; the latter in the natural algebraic view. In the former the projective properties of the two figures are the same; in the latter the two expressions of any projective property of the one figure are the same."

Scott (1894)

Certain Modern Ideas and Methods in Plane Analytical Geometry