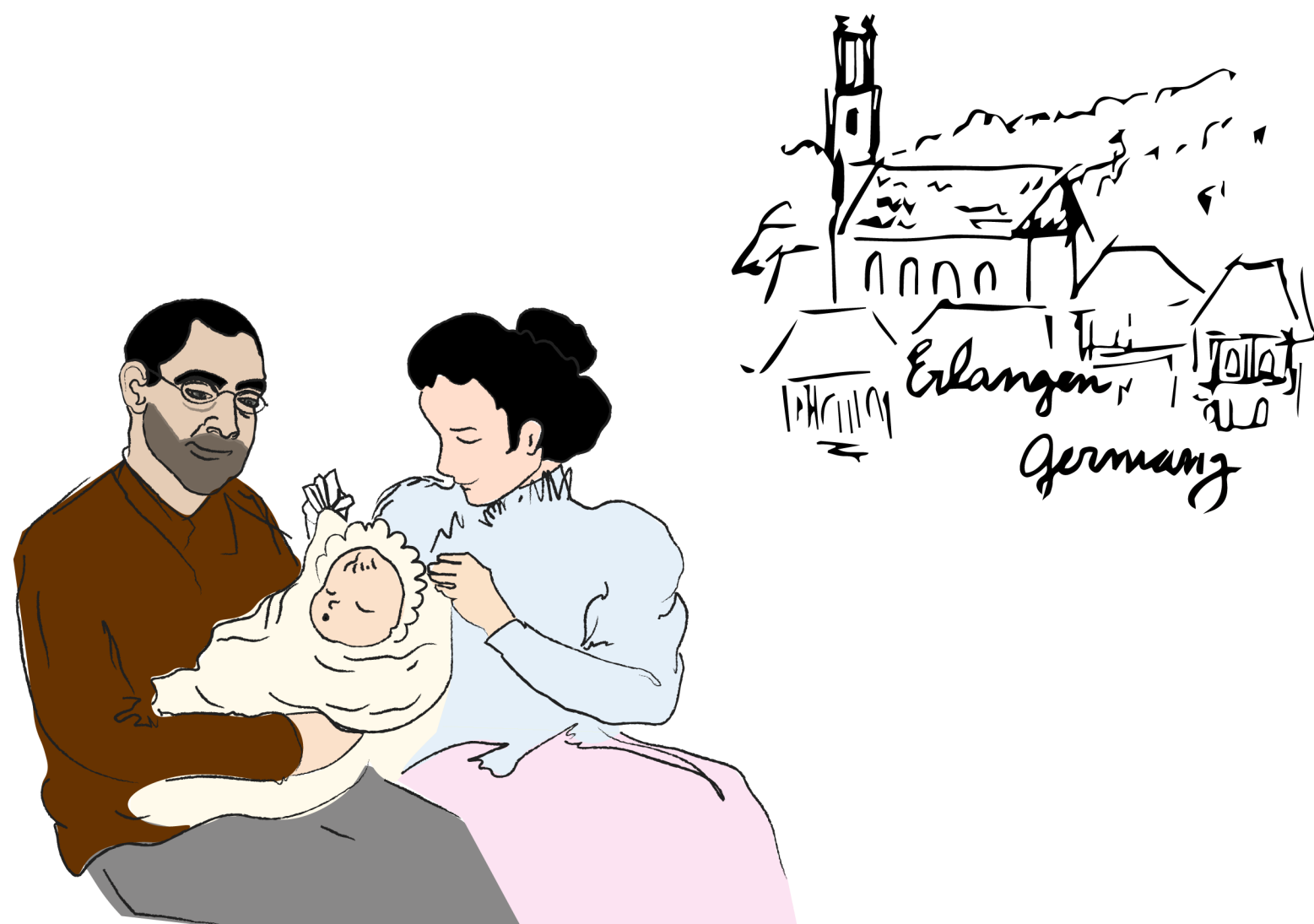


**Emmy Noether**

**The Mother of  
Modern Algebra**

**Ellie Wessinger  
Avery Bicks**

Emmy Noether was born to Max Noether and Ida Amalia Kaufmann in the provincial town of Erlangen, Germany on March 23, 1882.<sup>1, 2</sup> From the moment Emmy was born, she was destined for success.



Mathematics and education were ingrained in the Noether family's genetics. Max Noether worked as a mathematics professor at the local University. Two of Emmy's younger brothers, Fritz and Alfred, also grew up to be accomplished scientists.<sup>3, 4</sup> Little did anyone know that Emmy, the sole daughter of the family, would become the most renowned of all the Noether's.

Emmy's contributions to the field of abstract algebra and the greater mathematical subject are some of the most significant to date. However, as a woman trying to enter a male-dominated discipline, Emmy was met with many challenges and hurdles.

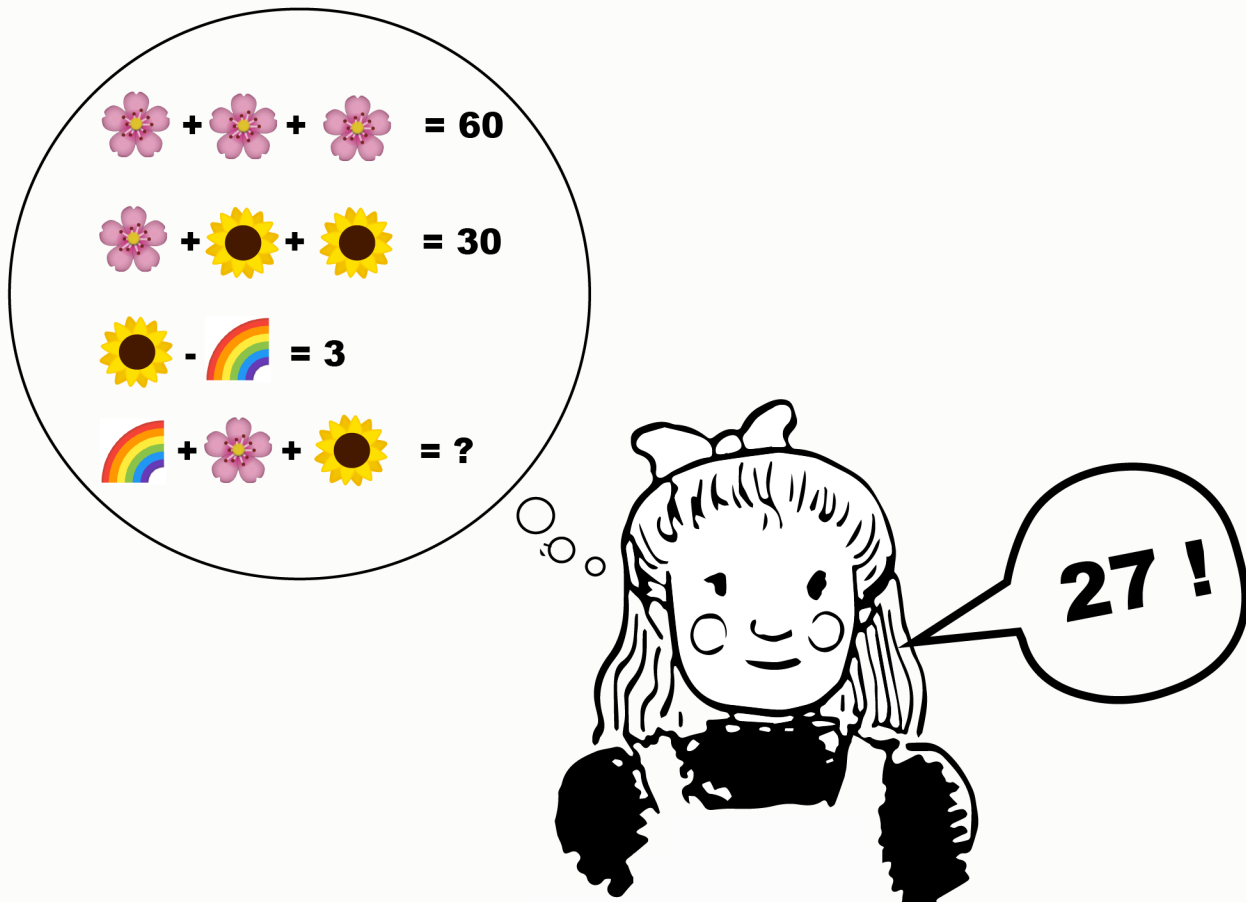
Emmy was a warm, gentle, and non-rebellious young girl. Growing up as the daughter of an upper-middle-class family in the late 19th century, Emmy had a stereotypical childhood. She learned domestic skills, such as cooking and cleaning. In her spare time, Emmy pursued dance and piano – a passion that likely stemmed from her mother, who was an accomplished pianist.



Between ages 7 and 15, Emmy attended the Städtische Höheren Töchterschule, a Municipal school for the higher education of girls. Outside of school, Emmy was often loud and unorganized. After learning to play the “Happy Farmer” song for beginners, Emmy quit piano lessons after realizing she was not as gifted of a pianist as her mother.<sup>5</sup> At this point, Emmy transitioned all her musical pursuits to dance.

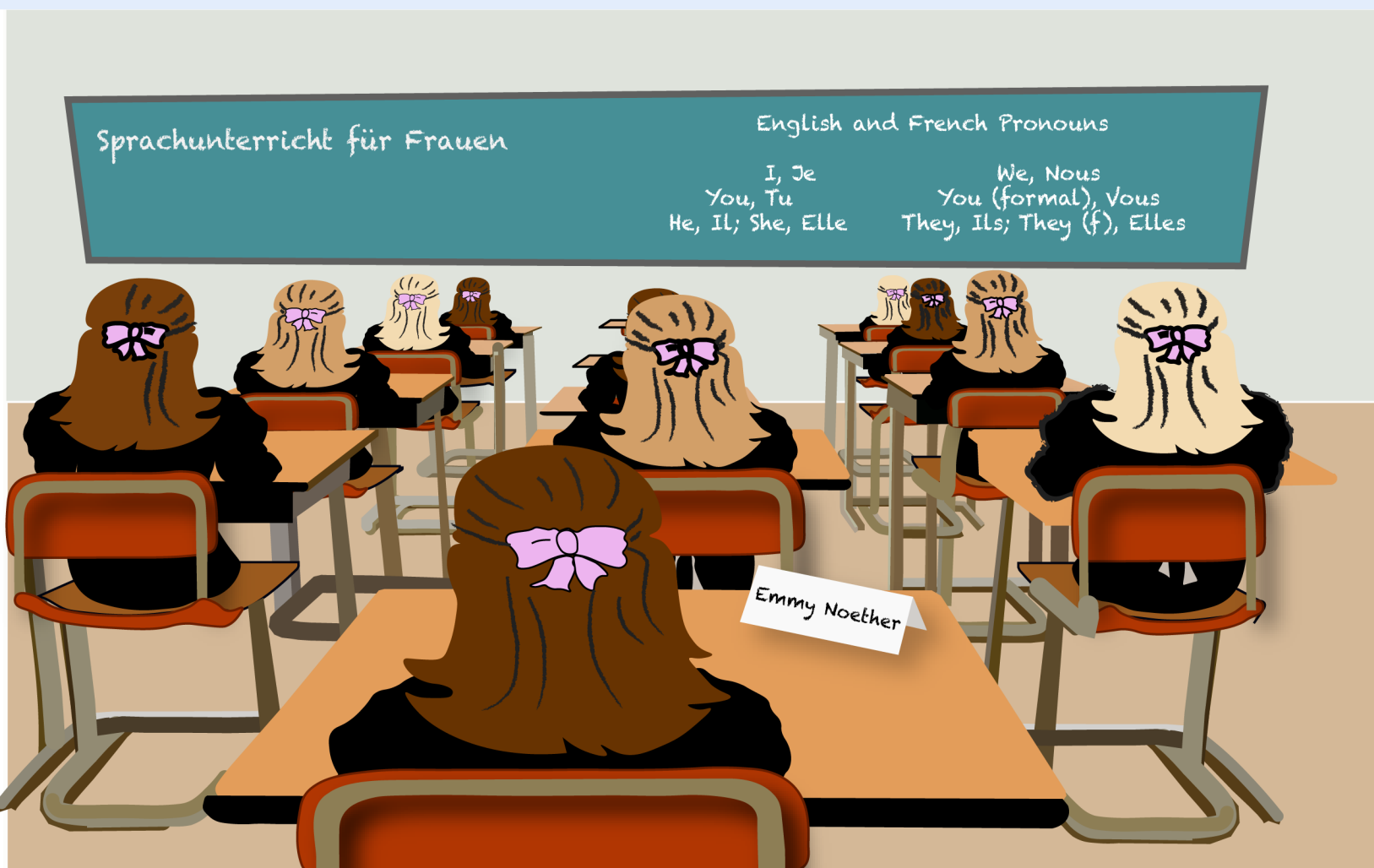
Throughout her childhood, Emmy often captivated her peers and elders with her mathematical aptitude and quick-mindedness.

One story goes that while Emmy was socializing with her friends at a children's birthday party, a supervising adult posed a complicated math problem. This adult told the children that "the sum of three pink flowers equals 60. The sum of one pink flower and two sunflowers equals 30. A rainbow subtracted from a sunflower equals 3. What is the sum of one rainbow, one pink flower, and one sunflower?"



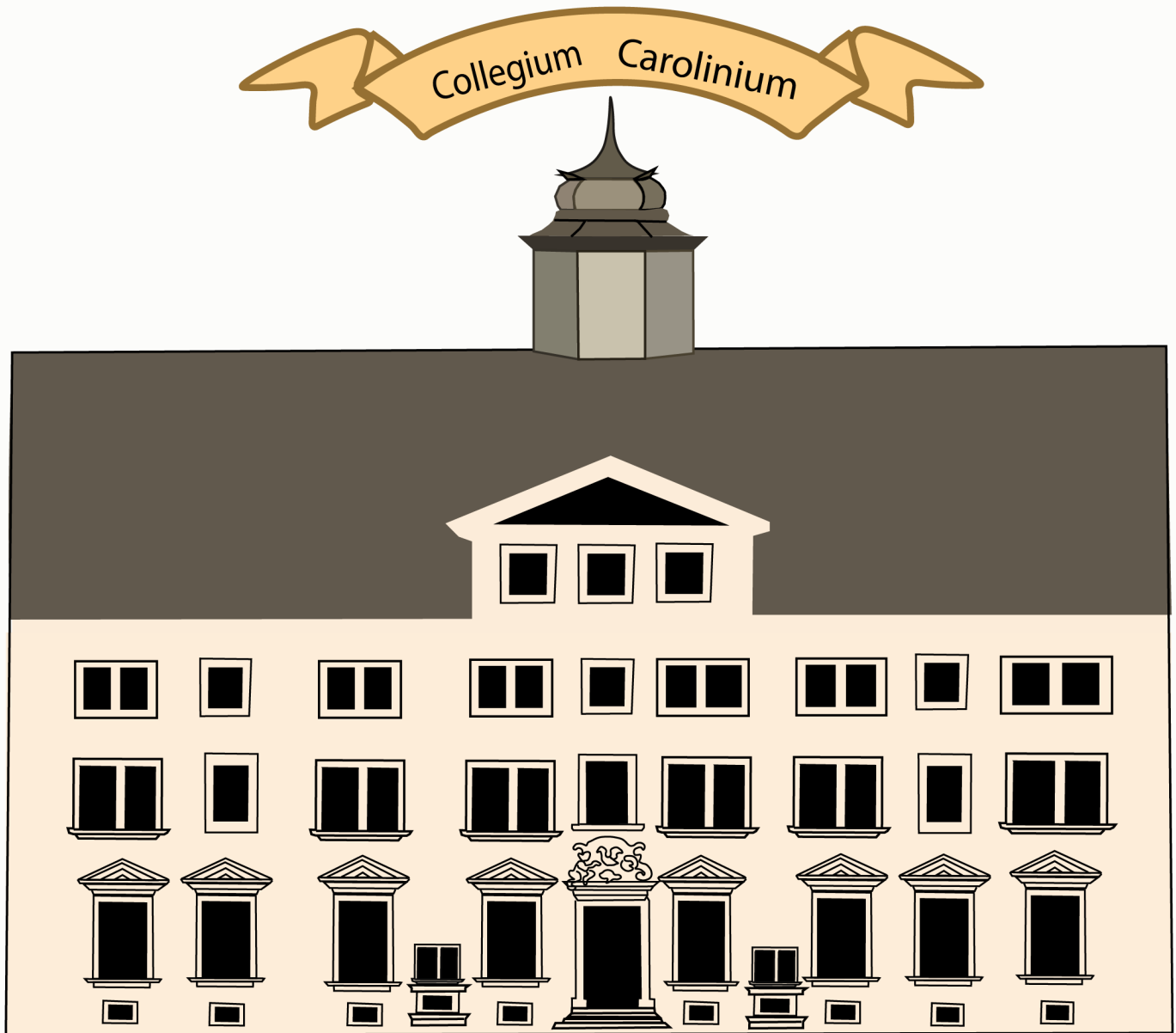
Emmy immediately responded "27!" Emmy's quick response baffled her peers and onlooking adults. "How could a young girl have solved such a complicated combinations problem in a matter of seconds?" they thought.

While Emmy was inspired by her father's profession and teachings, she was aware that given societal rules at the time, she would not be able to follow in her father's footsteps easily. So, instead, Emmy spent her teenage years studying the arts and language, and by age 18, she had become certified to teach French and English classes at all-girls schools. While Emmy enjoyed language and was ultimately interested in teaching, she wanted to continue her education at the university level.

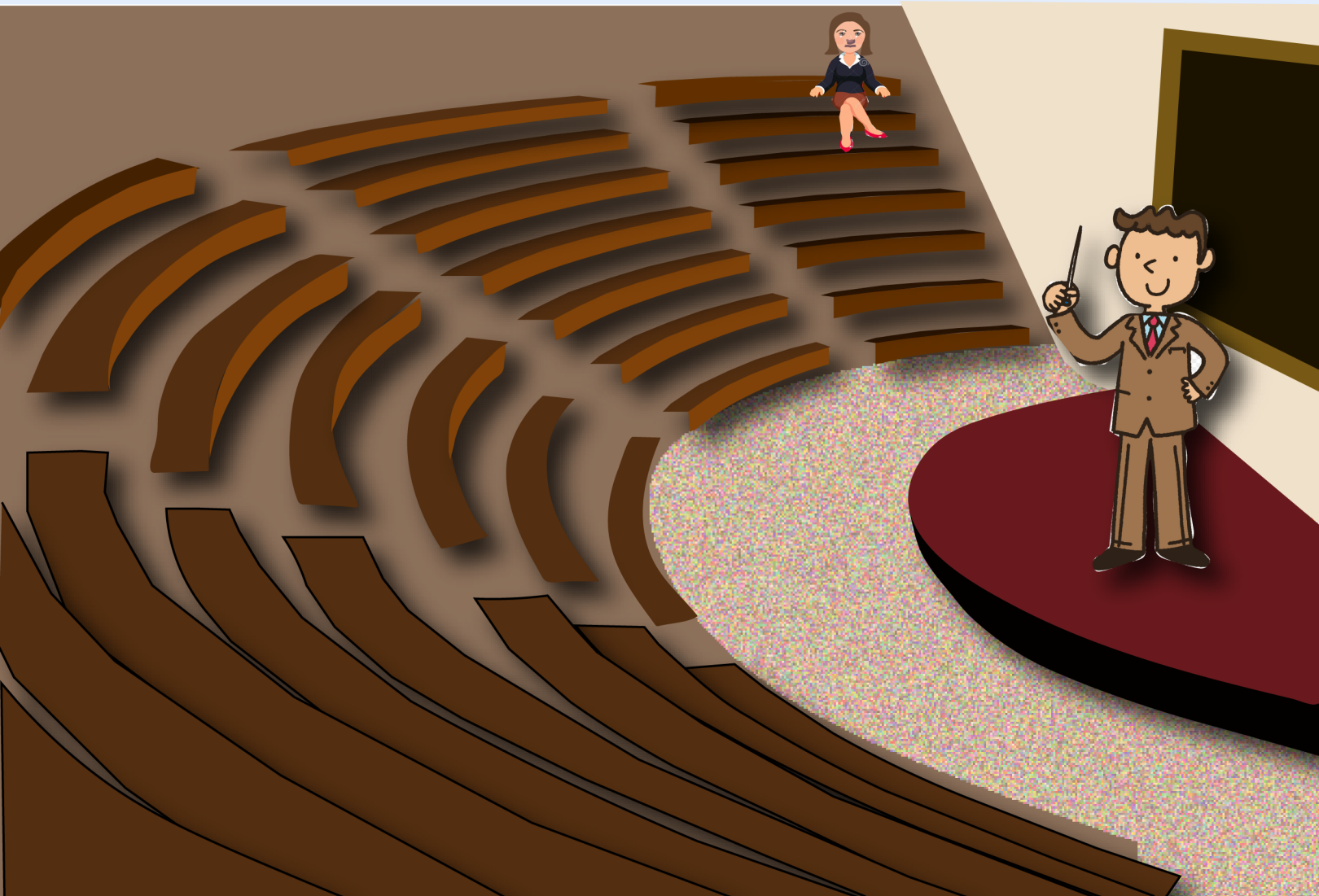


However, attending university as a young woman in the early 1900s proved to be a challenging undertaking. During the early 20th century, very few women attended universities in Germany. Women were seldom permitted to attend lectures or even to take the university entrance exams. At the time, women were often considered to be a distraction in an academic setting or unqualified.

Despite having all odds against her, Emmy was committed to pursuing a mathematical education at the University of Erlangen.<sup>6</sup> At first, Emmy was permitted only to audit classes. She was required to seek the permission of each professor whose course she wished to attend. Nevertheless, Emmy was dedicated to advance her studies, regardless of the prejudices and complications.

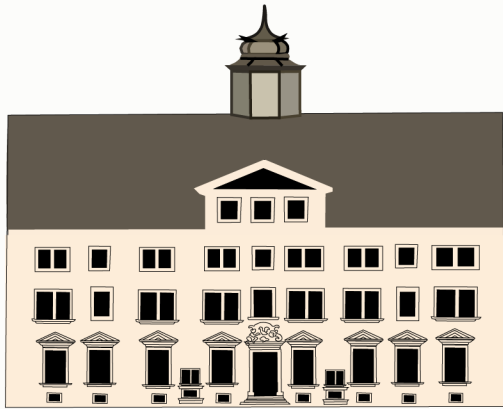


Luckily, since Emmy's father worked at the University of Erlangen, she was cordial with many professors on campus and received approval to audit their classes. Emmy attended courses at the University for two years. During this time, she was one of the few female students on campus.

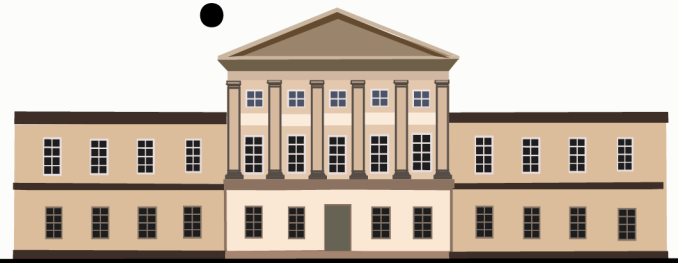


In fact, during the winter semester of 1900 Emmy was one of only three female auditors at the school, as compared with the 984 male students enrolled.

After two years of auditing lectures, Emmy completed the matura examinations and received a university certificate (known as a "Reifeprüfung"). After receiving her certificate, Emmy spent a semester at the University of Göttingen, where she audited lectures taught by famous mathematicians such as Felix Klein, David Hilbert, and Otto Blumenthal.<sup>7, 8, 9, 10</sup>



University of Erlangen

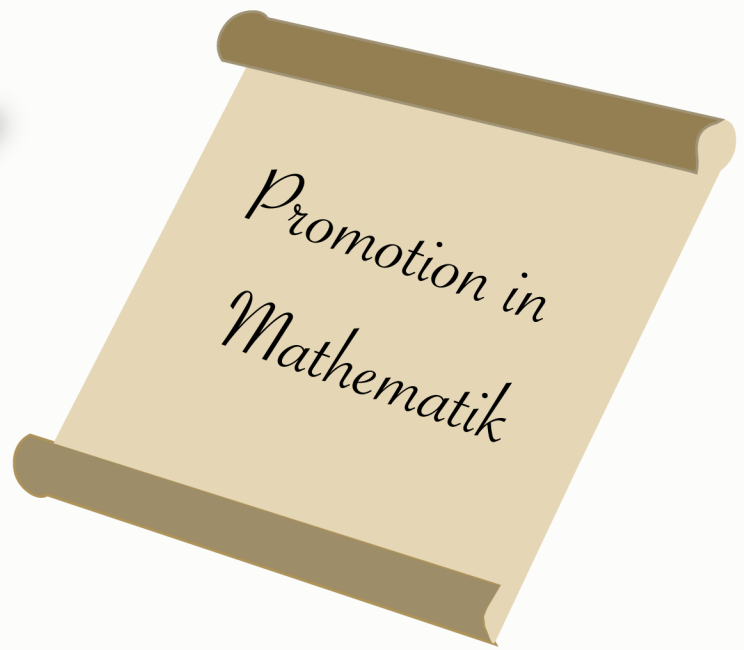


University of Göttingen

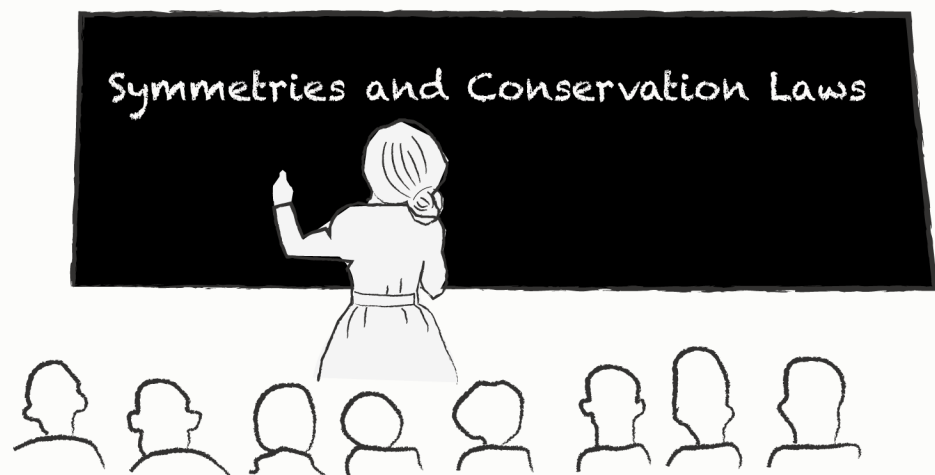
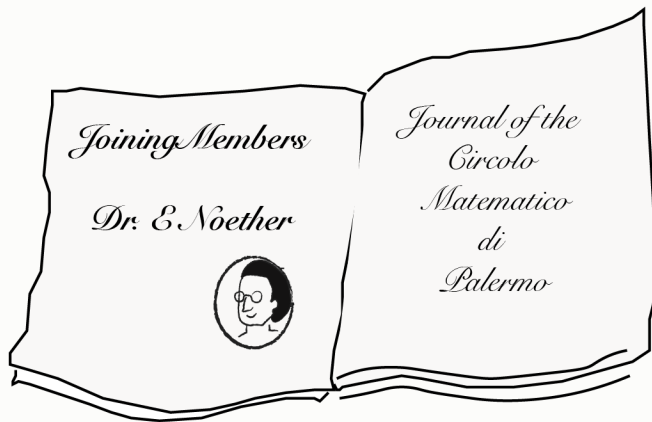
After a single semester at Göttingen, Emmy heard that the laws in Germany changed to allow female students to enroll in universities. Emmy was no longer required to attend classes as an auditor. Instead, she would be able to register and participate alongside her male counterparts. As a result, Emmy decided to return to Erlangen. In October 1904, she officially enrolled as a mathematics student at the University of Erlangen. Unsurprisingly, academic order was not disturbed with the presence of women in the classroom.



After three years, Emmy culminated her studies at the University with a lengthy paper known as a doctoral dissertation. Emmy's paper was based on the theory of invariants and was published in Crelle's Journal, officially marking the young woman's first publication.<sup>11</sup> In December 1907, Emmy received her Ph.D. in mathematics from Erlangen and was awarded her degree with the highest distinction.

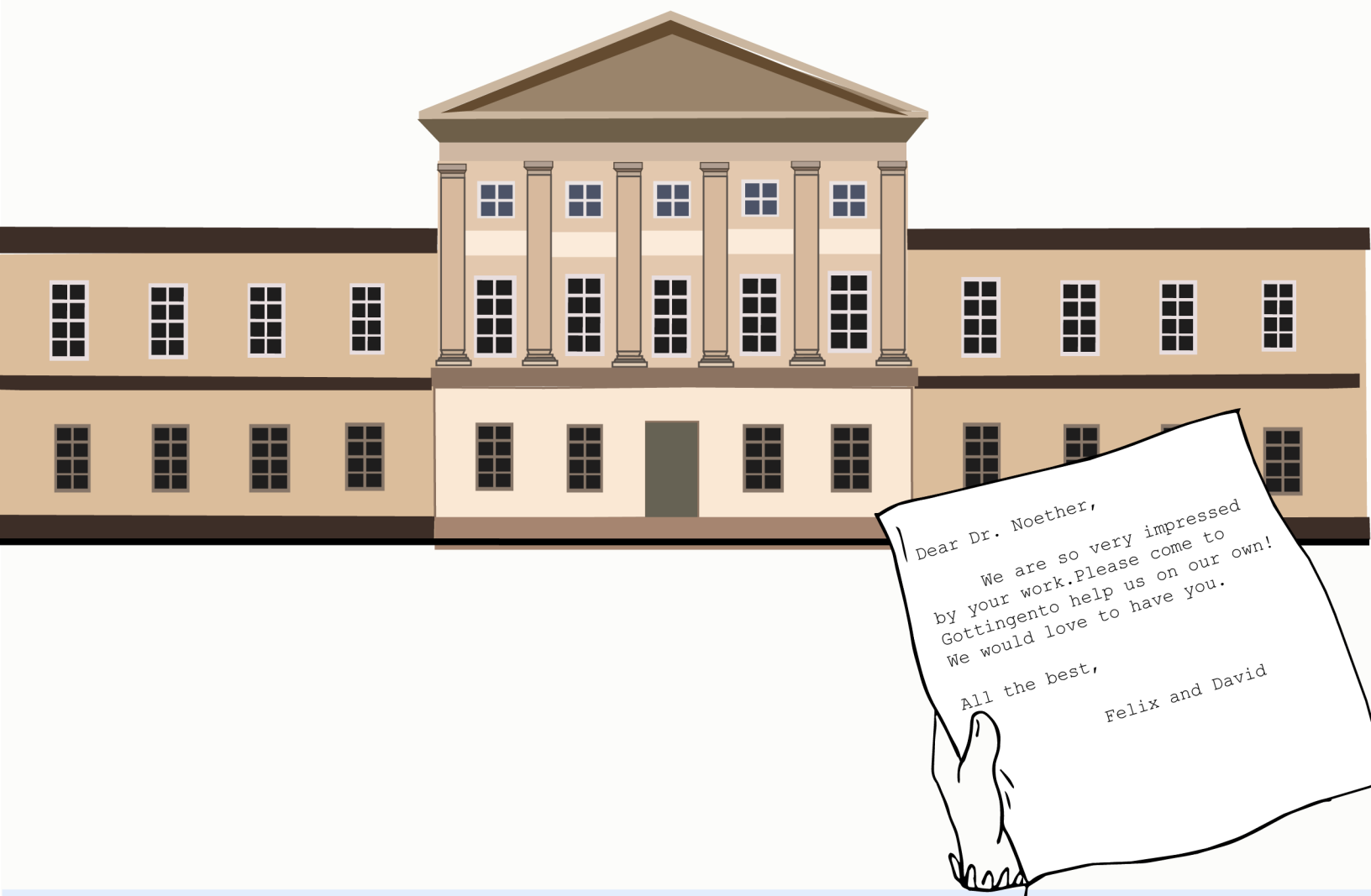


After completing her Ph.D., Emmy remained in Erlangen where she worked at the local mathematical institute and assisted her aging father. While at the institute, Emmy carried out her research and occasionally assisted her father in his lectures at the University.



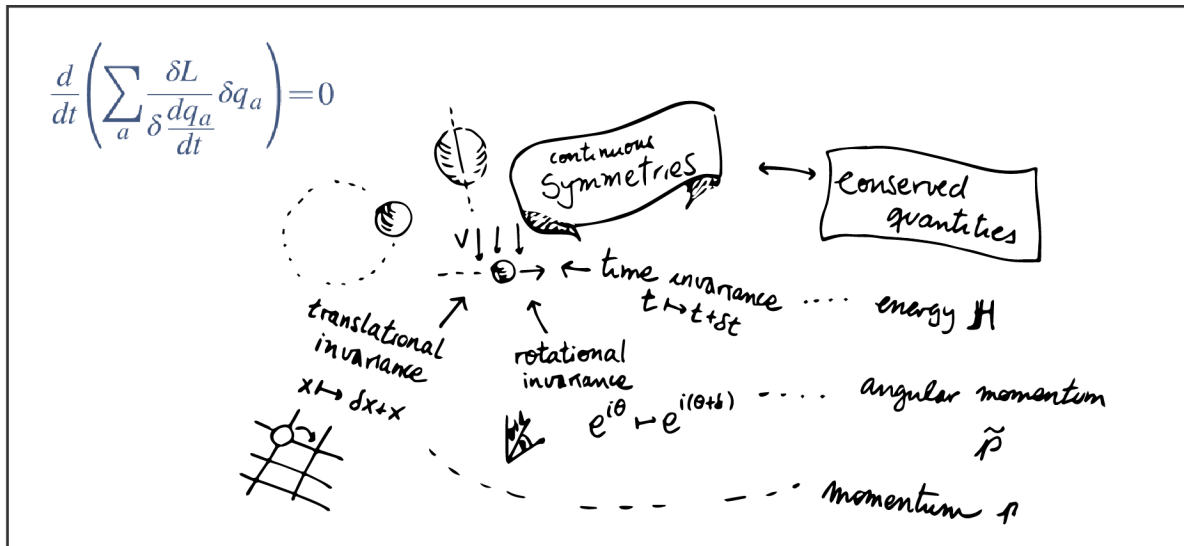
In 1909, Emmy became a member of the German Association of Mathematicians.<sup>12</sup> This Association offered a valuable forum, where young mathematicians could share research and learn about new areas in the field of mathematics during annual meetings. Emmy loved these meetings! Not only was she able to present her research on invariants, rational functions, and ideal theory, but also she was able to meet new mathematicians. These annual congregations ultimately solidified Emmy's passion for mathematics and the study of invariants in particular.

In 1915, Hilbert and Klein were in the midst of their work on Einstein's theory of general relativity at the University of Göttingen. In the course of their research, David Hilbert and Felix Klein had arrived at a crossroads. They had discovered an inconsistency in Einstein's theory, which they were not able to reconcile. Already, Emmy's intelligence was both known and appreciated by a significant portion of the scientific community. As a result, Hilbert and Klein solicited Emmy's assistance.

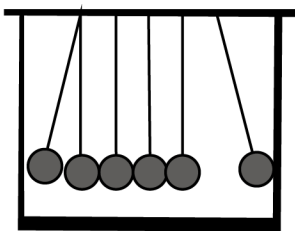


In the Spring of 1915, Emmy joined the two mathematicians at the University of Göttingen without a regular job. Emmy's brilliance made her a delight and an honor to work with closely. Noether deployed her expertise in abstract algebra, which Hilbert and Klein believed to be critical to the field of relativity. After only months of work, Emmy successfully resolved the roadblock impeding the scientists.

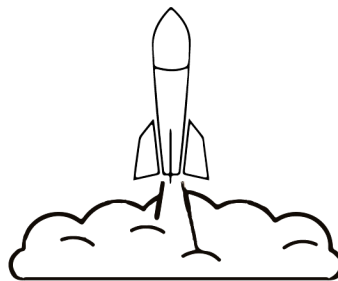
Emmy's discovery, formally known as Noether's Theorem, expresses the equivalence that exists between the laws of conservation and the symmetry invariance of the Lagrangian of a physical system.<sup>13</sup> This explains the conservation of the moment of inertia. Through publishing this theorem, Emmy had rescued the theory of relativity.



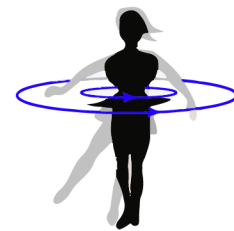
Laws of Physics are symmetric in space, time and rotation. Noether's theorem suggests that due to these symmetries, momentum, energy, and angular momentum are conserved.



When a swinging ball hits a row of balls, the ball on the other end flings out. As a result, total momentum is conserved. This is known as symmetry of space.



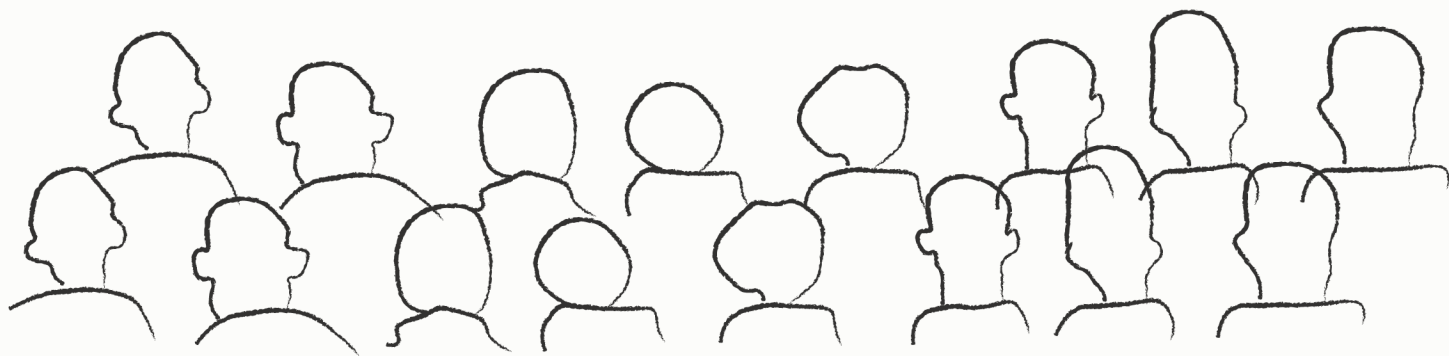
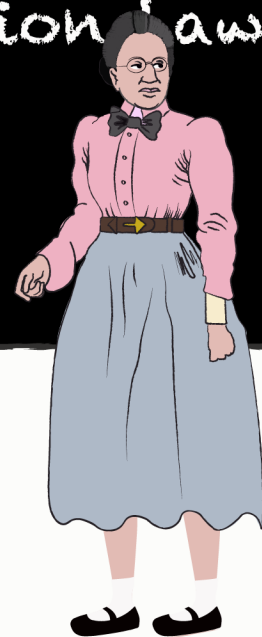
A rocket launch converts chemical energy into potential energy. Total energy stays constant due to the symmetry of time.



The speed of a twirl increases as it tightens. This is because total angular momentum remains constant due to the symmetry of rotation.

From 1915-1919, Emmy worked as an assisting lecturer to Hilbert while simultaneously pursuing her own research. During this time, significant political structure changes occurred in Germany. As a result, Emmy was granted professorship status at the University and began teaching her own lectures.

## Symmetries and Conservation Laws

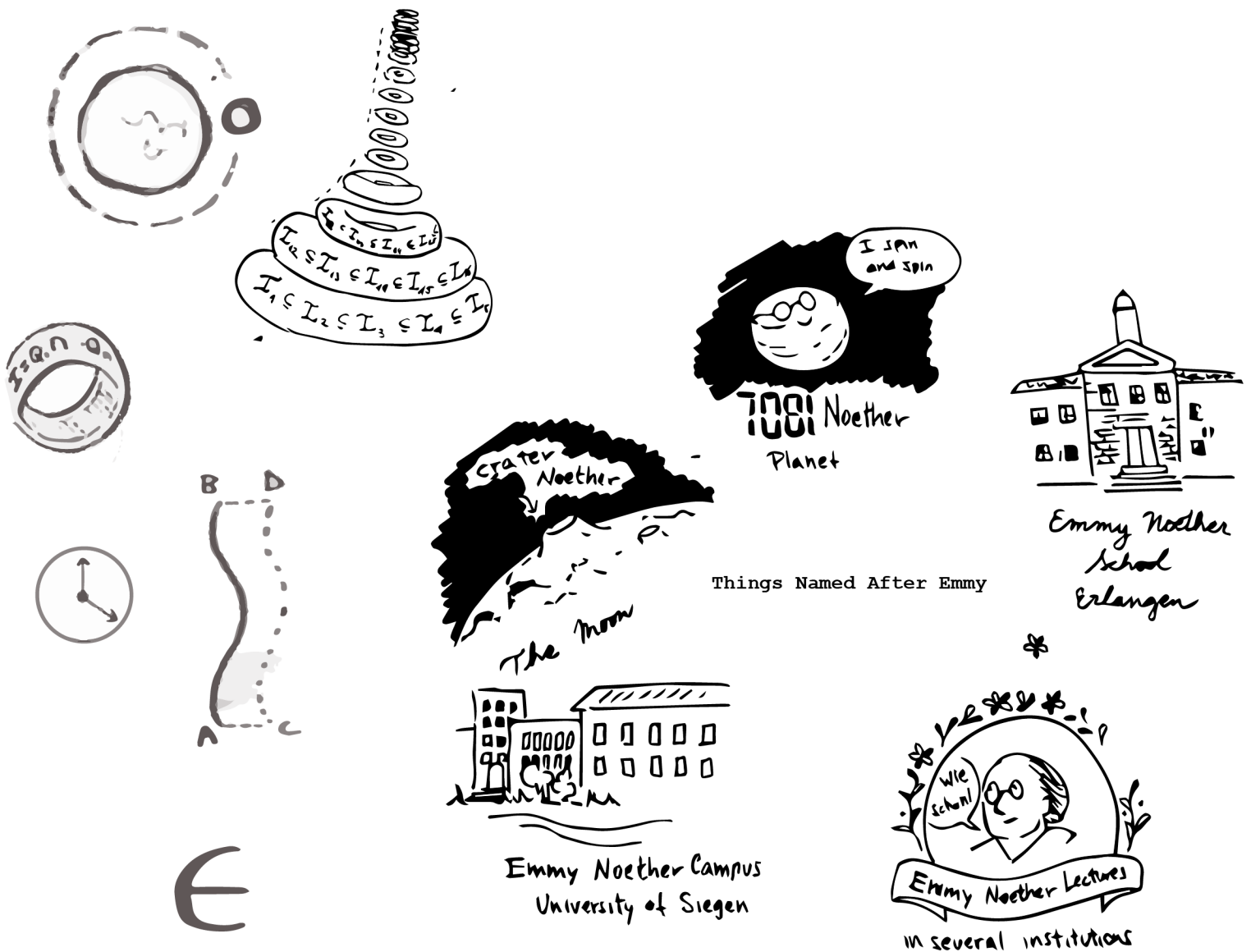


The rise of Adolf Hitler had a significant influence on the trajectory of Noether's career. Like all Jews living in Nazi Germany during the 1930s, Noether was lucky to escape alive. In 1933, she was removed from her position at the University of Göttingen, as she was deemed disloyal to the German Reich.<sup>14</sup>



With bravery and grace, Noether safely left Germany and became a refugee scholar in the United States. In America, Noether accepted a job at Bryn Mawr College, a women's liberal arts college in Pennsylvania.<sup>15</sup> Her brilliance and contributions were well known in the United States, which resulted in several invitations to serve as a guest lecturer. In 1934, Noether started lecturing in New Jersey. Interestingly, she was never allowed to teach or lecture at Princeton University (NJ), because it was still an all-male institution.

Emmy spent her remaining years at Bryn Mawr where she taught and continued to publish her work. Today, Emmy is recognized for her work in ring theory, group theory, conservation of angular momentum, and continuous symmetries. During her career, Emmy was highly esteemed among her peers, and her legacy continues to inspire aspiring female mathematicians today.



## Endnotes

1. *Max Noether:* Max Noether was Emmy Noether's father. He was born in Mannheim in 1844. At a young age, he contracted polio, which he lived with through his adult years. He studied mathematics at the University of Heidelberg in the late 1860s. Afterward, he moved to the University of Erlangen, where he helped develop the field of algebraic geometry.

2. *Ida Amalia Kaufmann:* Ida Amalia Kaufmann was Emmy Noether's mother. Ida was born in 1852 to a wealthy Jewish merchant family. Within the Noether family, Ida assumed a domestic role. She was also known to play piano in her spare time.

3. *Alfred Noether:* Alfred Noether was the oldest son of the Noether family. He was born in 1883. He studied chemistry at the University of Erlangen and received his doctorate in 1909. Alfred died in 1918, only nine years later.

4. *Fritz Noether:* Fritz Noether was Emmy Noether's second oldest brother. He was born in 1884. At the University of Munich, Fritz studied applied mathematics and received his doctorate under the well known Aurel Voss in 1909. Fritz conducted his postdoctoral studies in Göttingen. Fritz later moved to Karlsruhe where he worked as an assistant at the Technische Hochschule. In the summer of 1914, Fritz worked in the army. He then returned to Technische Hochschule briefly before becoming the second chair of Higher Mathematics and Mechanics at the University of Breslau. In November 1937, Fritz was arrested in the Russian city of Tomsk for working as a German spy and spreading anti-soviet propaganda. He was later sentenced to death.



## Endnotes

5. *Happy Farmer Son*: The *Happy Farmer* song ("Fröhlicher Landmann, von der Arbeit zurückkehrend") was a piece composed by the German pianist Robert Schumann in 1848 for his three daughters. This piece, along with the rest of Schumann's *Album for the Young*, was intended to be played by young children or beginners.

6. *University of Erlangen*: The University of Erlangen is a public research university in Erlangen, Germany. The university was founded in 1742. Some notable alumni and professors who attended the university include Emmy Noether, Max Noether, Felix Klein, Karl von Hegel, Hans Geiger, and Ludwig Erhard.

7. *University of Göttingen*: University of Göttingen is a public research university in Göttingen, Germany. The university was founded in 1737 by George II of England. In the late 19th century, the university's Mathematical Institute attracted accomplished mathematicians from across the world. Carl Friedrich Gauss, Bernhard Riemann, and David Hilbert all worked at the university at various times. Over 45 Nobel Prizes have been awarded to individuals associated with the University to date.

8. *Felix Klein*: Felix Klein was a prominent German mathematician who is primarily known for his work in group theory, non-Euclidean geometry, complex analysis, and geometry. Felix Klein was born in 1849. He received his doctorate from the University of Bonn in 1868. Throughout his career, Klein taught at numerous schools including the University of Erlangen, the Technische Hochschule in Munich, Leipzig University, and the University of Göttingen.

## Endnotes

9. *David Hilbert*: David Hilbert was a well-known German mathematician. David Hilbert was born in 1862 in Prussia. He attended the University of Königsberg and completed his dissertation in 1884. After graduating, Hilbert taught at Königsberg. He later moved to Göttingen where he worked as a professor until retirement. He is known for his work on invariant theory, algebra, algebraic number theory, geometry, and proof theory.

10. *Otto Blumenthal*: A prominent German mathematician that lived from 1876–1944. He attended Göttingen and wrote his thesis as David Hilbert's advisee. Similar to Noether, Blumenthal was forced out of Germany in 1933 due to her Jewish heritage. His life came to a tragic end in a concentration camp in 1944.

11. *Crelle's Journal*: Crelle's Journal is a prominent mathematical journal, which began publishing works in 1826. The Journal has published numerous notable works from mathematicians such as Emmy Noether, Georg Cantor, and Carl Friedrich Gauss.

12. *The German Association of Mathematics*: The prominent mathematical society in Germany. David Hilbert and Felix Klein are two of the institution's most notable founders. Max Noether, Emmy's father, was the president of the Association in 1899. In addition to publishing works and offering an annual forum for discussion, the Association awards the Cantor Medal. The award, named after the Association's first president Georg Cantor, is presented to outstanding German Mathematicians.

13. *Noether's Theorem*: Expresses the equivalence that exists between the laws of conservation and the symmetry invariance of the Lagrangian of a physical system. Emmy's discovery of this theorem was pivotal to solving the issues with Einstein's Theory of Relativity. Noether's Theorem explains the conservation of the moment of inertia.

## Endnotes

14. *German Reich*: The German Reich refers to that nation-state of Germany, which existed from 1871-1945. The era of the Reich explored in our text is referred to as the "Third Reich." This was the era of Nazi control that lasted from 1933 until 1945. During the Third Reich, Emmy was forced to evacuate Germany and seek refuge in the United States due to her religion.

15. *Bryn Mawr College*: Bryn Mawr College is an women's liberal arts college located in Pennsylvania. This institution was Emmy's landing place in the United States. Emmy accepted a job as a professor at Bryn Mawr. Emmy's remains can be found in the Cloisters of Bryn Mawr.

## Sources

In 1980, the Australian historian Auguste Dick published a comprehensive biography on Emmy Noether.

Auguste Dick, *Emmy Noether, 1882-1935*, (published in German in 1971), translation by H. I. Blocher, Birkhauser, Boston, 1981.

A primary source which includes a translated copy of Noether's Theorem.

Noether, E. (n.d.). *Invariant Variations Problems*. Retrieved from <http://cwp.library.ucla.edu/articles/noether.trans/english/mort186.html>

This source provides a brief biography on Noether, includes Noether's obituaries and copies of her work.

Noether, Emmy. *Emmy Noether : A Tribute to Her Life and Work*. Edited by James W Brewer and Martha K Smith, M. Dekker, 1981.

This source provides a comprehensive collection of Noether's works on both mathematics and physics.

Teicher, Mina. *The Heritage of Emmy Noether*. Gelbart Research Institute for Mathematical Sciences and the Emmy Noether Research Institute, Bar-Ilan University, 1999.

This source includes a contemporary view of Noether's work and includes some illustrations which we incorporated on page 14.

Cavna, M. (2015, March 23). *Emmy Noether Google Doodle: Why Einstein called her a 'creative mathematical genius'*. Retrieved from [https://www.washingtonpost.com/news/comic-riffs/wp/2015/03/23/emmy-noether-google-doodle-why-einstein-called-her-a-creative-mathematical-genius/?utm\\_term=.ba4f1fbcca50](https://www.washingtonpost.com/news/comic-riffs/wp/2015/03/23/emmy-noether-google-doodle-why-einstein-called-her-a-creative-mathematical-genius/?utm_term=.ba4f1fbcca50)

This brief article by Kennedy Steve provided us with a general view of Noether's life and helped us determine which areas of Noether's history should be highlighted in our text.

KENNEDY, STEVE. "Emmy Noether." *Math Horizons*, vol. 4, no. 2, 1996, pp. 17-17. [https://www-jstor-org.ccl.idm.oclc.org/stable/25678088?seq=1#metadata\\_info\\_tab\\_contents](https://www-jstor-org.ccl.idm.oclc.org/stable/25678088?seq=1#metadata_info_tab_contents)

This scholarly source provides details about Noether's meeting with Felix Klein and David Hilbert and inspired our decision to focus on Noether's Theorem.

Kosmann-Schwarzbach, Yvette. *The Noether Theorems : Invariance and Conservation Laws in the 20th Century*. Springer, 2011.

The following source is a children's book which provides an embellished perception of Emmy's life. This source inspired our decision to include the anecdote about Emmy's ability to solve complicated combination problems. Both Augustine Dick and M W B Tent both claim that Emmy had solved a complicated combinations problem at a children's party, the details of such problem are unknown. For this reason, we included our own combinations problem in our book.

Tent, M. B. W. *Emmy Noether : The Mother of Modern Algebra*. A.K. Peters, 2008.

Frl. Noether an K. Klein 22 B. Erlangen, 1915. 1A

Ihre wertvolle Frage betreffend:

Ich danke Ihnen sehr für die Zusendung Ihrer Note, und für Ihre Mitteilung über Runge's Problem. Die Längere Reflexion bin ich wohl damit beschäftigt, es möglichst vollständig gerade im allgemeinsten Fall dem der Allgemeinheit leichter Ordnung, allgemein, wenn man einen gewissen Variationsproblem  $\int (f(x, \frac{dy}{dx}) - y) dx$  in 2 Stellen fest über die Endbedingung  $\int y \cdot \frac{dy}{dx} = 0$  (oder  $c_1$ ) erfüllt, ist es gerade die Lagrange'sche Differentialgleichung, ist man kann auf keine Weise einen Lagrange'schen Ansatz, ist es dann, man positiv ist an Stelle der Runge'schen

Bedingung für Lagrange'sche Methode der kleinsten Wirkung

$$\int \left( \sqrt{\left(\frac{dy}{dx}\right)^2 + 1} - \left(\frac{dy}{dx}\right)^2 dx = 0 \right)$$

für welche an Stelle der Lagrange'schen Gleichung die Lagrange'sche Gleichung:

$$\int y \cdot \frac{dy}{dx} = 0 \quad \text{[entsprechend Variation gegenüber } t = 0 \text{, 2 Stellen]}$$

man kommt oft auf die gewöhnlichen Gleichungen  $\frac{dy}{dx} = 0$  gerückt, wenn man den Lagrange'schen Ansatz allmählich in die gewöhnliche Lagrange'sche Gleichung für unendliche Folge von Stellen - Und man wird die Methode so weit, wobei die Variationsform, dass man das "unendliche" Prinzip der kleinsten Wirkung an die Stelle stellt, und wie das "Methode" nennt man das folgt, so heißt das "Methode" kann allgemein

Emmy Noether's Letter to Felix Klein, March 1918