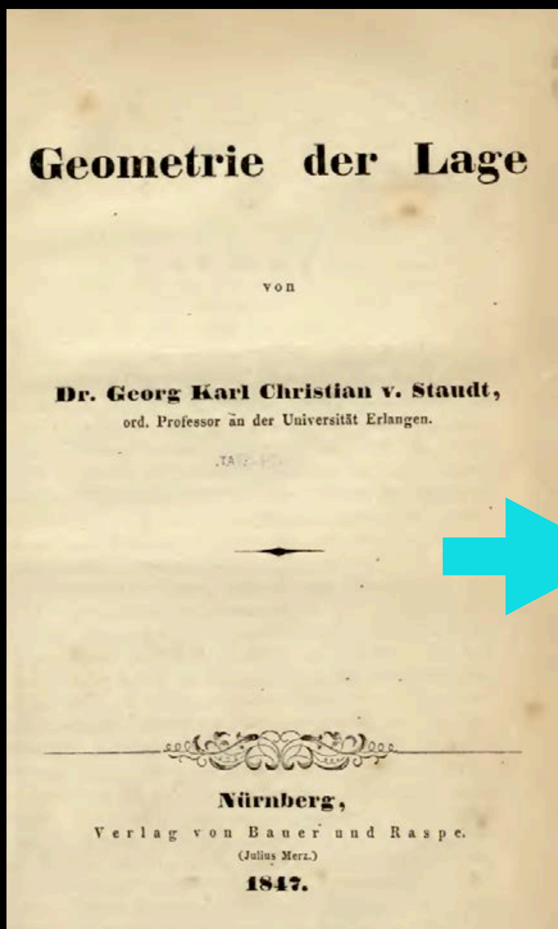
# Russell, von Staudt, and Hilbert in the Bryn Mawr College Mathematics Journal Club

Seminar PhilMath Intersem 2022
"Logic and Geometry"
Jemma Lorenat, Pitzer College
jlorenat@pitzer.edu



Geometrie der Lage

von

Dr. Georg Karl Christian v. Staudt,

ord. Professor an der Universität Erlangen.

Nürnberg,

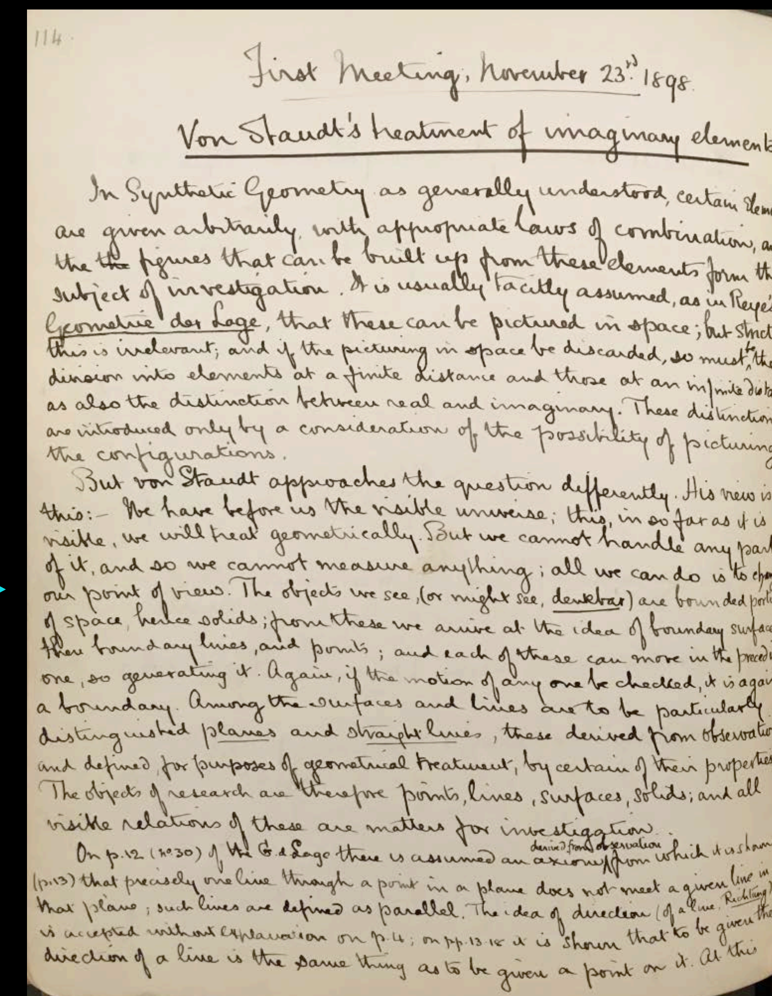
Verlag von Bauer und Raspe.

(Julius Merz.)

1847.

Texts on pure geometry

Bryn Mawr Mathematics Department



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First Meeting, November 23rd 1898

Von Staudt's treatment of imaginary elements.

In Synthetic Geometry as generally understood, certain elements are given arbitrarily, with appropriate laws of combination, and the ~~the~~ figures that can be built up from these elements form the subject of investigation. It is usually tacitly assumed, as in Reye's Geometrie der Lage, that these can be pictured in space; but strictly this is irrelevant; and if the picturing in space be discarded, so must be the division into elements at a finite distance and those at an infinite distance, as also the distinction between real and imaginary. These distinctions are introduced only by a consideration of the possibility of picturing the configurations.

But von Staudt approaches the question differently. His view is this:— We have before us the visible universe; this, in so far as it is visible, we will treat geometrically. But we cannot handle any part of it, and so we cannot measure anything; all we can do is to change our point of view. The objects we see, (or might see, denkbar) are bounded portions of space, hence solids; from these we arrive at the idea of boundary surfaces, then boundary lines, and points; and each of these can move in the preceding one, so generating it. Again, if the motion of any one be checked, it is again a boundary. Among the surfaces and lines are to be particularly distinguished planes and straight lines, these derived from observation, and defined for purposes of geometrical treatment, by certain of their properties. The objects of research are therefore points, lines, surfaces, solids; and all visible relations of these are matters for investigation.

On p.12 (No 30) of the G. d. Lage there is assumed an axiom derived from observation from which it is shown (p.13) that precisely one line through a point in a plane does not meet a given line in that plane; such lines are defined as parallel. The idea of direction (of a line, Richtung) is accepted without explanation on p.4; on pp.13.14 it is shown that to be given the direction of a line is the same thing as to be given a point on it. At this

Journal Club Notebooks

For success, beyond all things, college teaching in our subject must be alive; it consists in the transference and impress of thought, not of facts. Uniformity belongs to dried specimens, not to living organic growth. […] By sheer force of personality the teacher must oblige his class to receive his instruction; and if he has this in him, it is his own remembered experience as a student supplemented by his increasing experience as a teacher that brings success.

Charlotte Angas Scott (1910)
The Preparation of College and University Instructors in Mathematics
*Bryn Mawr Mathematics Journal Club Notebooks (1910–1911)*

How do mathematics students become mathematics researchers?

This talk will focus on a partial, local answer to this question with respect to the teaching of pure geometry to graduate students at Bryn Mawr College as documented in the Mathematics Journal Club Notebooks between 1896 and 1903.

In particular, I have two aims in this talk:

1. Probe how iconic texts/authors filtered through the local knowledge traditions of Bryn Mawr College.

2. Complicate the division between philosophical and pedagogical motivations in the foundations of geometry at the end of the nineteenth century.

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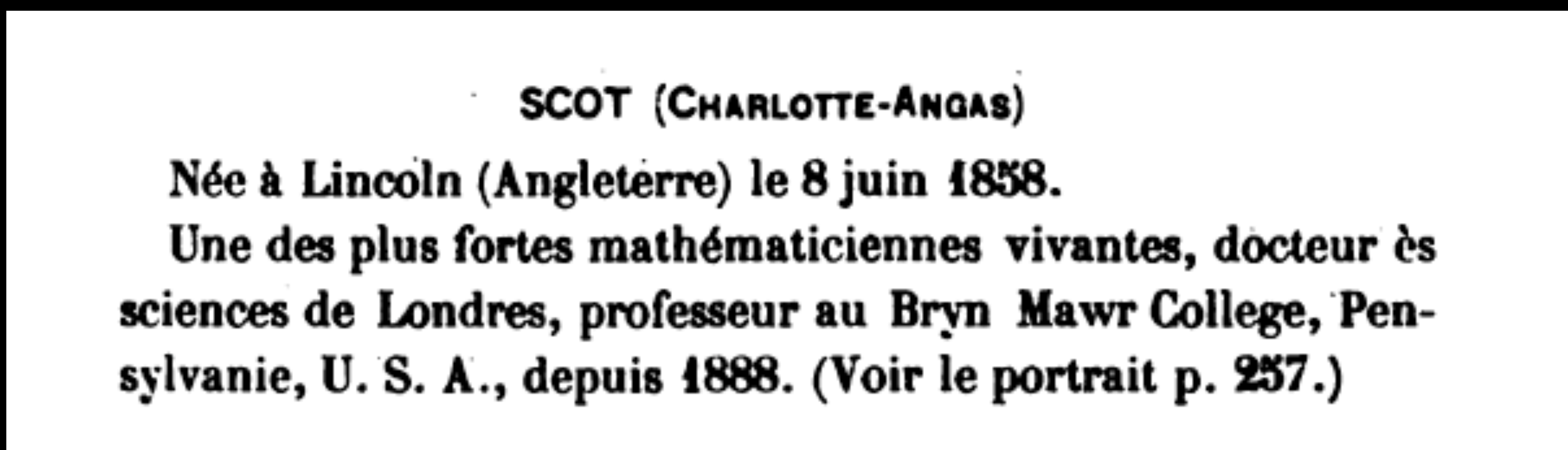
6. Imaginaries part III : all of the above
7. Conclusions

Russell, von Staudt, and Hilbert in the Bryn Mawr College Mathematics Journal Club

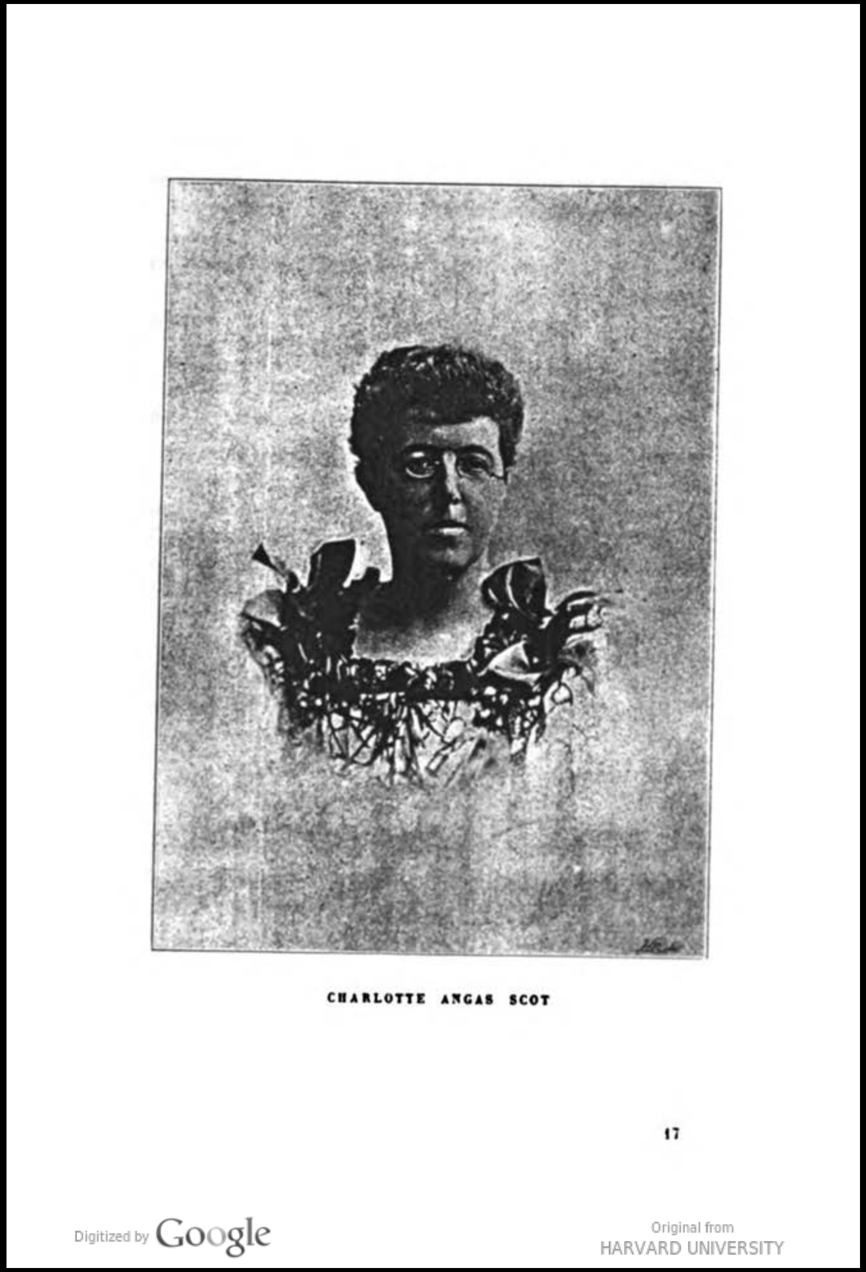
1. Charlotte Angas Scott and Bryn Mawr College
    A. Who was Charlotte Angas Scott?
    B. What did she publish?
    C. Research characteristics


SCOT (CHARLOTTE-ANGAS)

Née à Lincoln (Angleterre) le 8 juin 1858.

Une des plus fortes mathématiciennes vivantes, docteur ès sciences de Londres, professeur au Bryn Mawr College, Pensylvanie, U. S. A., depuis 1888. (Voir le portrait p. 257.)






A. Rebière (1897)
*Les Femmes dans la Science*

1. Charlotte Angas Scott and Bryn Mawr College

    A. Who was Charlotte Angas Scott?

    B. What did she publish?

    C. Research characteristics



*The binomial equation* $x^p - 1$; 1886, pp. 261-4 (Americ. journ. of math.).

*On the numerical characteristic of a cubic curve*; 1890.

*On the higher singularities of plane algebraic curves*, 1892, pp. 301-25 (idem).

*The nature and effect of singularities of plane algebraic curves*. 1893, pp. 221-43 (idem).

*On plane cubics*. 1894, pp. 247-77 (Royal soc., Philos. transact).

*An introductury account of certain modern ideas and methods in plane geometry*. New-York, 1894, in-8, pp. XII, puis 288.

*Arthur Cayley*. New-York, 1895, pp. 130-5. (Americ. math. soc.)

*Note on equianharmonic cubics*. 1895, pp. 180-5. (Messenger of math.).

*The three great problems of antiquity considered in the light of modern mathematical research*. New-York, 1896, pp. 157-64. (Americ. math. soc.).

Collaboratrice de *Educational times*, lectrice, conférencière, etc.

1. Charlotte Angas Scott and Bryn Mawr College
    A. Who was Charlotte Angas Scott?
    B. What did she publish?
    C. Research characteristics

Scott's research in analytical geometry characterized by (1) a dedication to interpreting algebraic results geometrically, (2) an attention to the relationship between appearance and reality, (3) technical skill in the accurate tracing of curves.

These characteristics are not only present in Scott's research, but also the dissertations of her graduate students.

But, not all of them translate well into "pure geometry" — there is no corresponding algebra and there are no drawn curves in what follows.

So will be considering how Scott's attention to the relationship between appearance and reality informs her reading in pure geometry and evolves as a result of teaching pure geometry.

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7. Conclusions

"to receive reports on special topics and listen to outline accounts of interesting theories that do not naturally present themselves in the regular graduate work"

The Mathematics Journal Club

A. Models

B. Texts

C. Bertrand Russell's 1896 lectures

*Are there any mathematical seminaries or clubs, and how are they conducted?*

All answered in the negative, except the following:

15. No clubs, unless special classes for voluntary and outside reading be so designated. Such classes are conducted like all other classes.

38. A club. The meetings of the club occur on alternate Tuesdays. Membership about 25; topics are assigned to or chosen by the student at his option; assistance is given him as he may need. The work is pedagogical, rather than original.

41. One. Reading and exposition of the more difficult parts of Williamson's Calculus.

51. In connection with the Science Club; by lectures.

67. There is a mathematical society, in which there is free choice of subjects for communication, and there are two or three seminaries for post-graduate students, conducted by the teacher on special lines.

MATHEMATICAL TEACHING AT THE PRESENT TIME. 303

*Are there any mathematical seminaries or clubs, and how are they conducted?*—Continued.

115. Yes, one. It proposes to discuss the literature of mathematics, to solve problems given by members, and to make original investigations.

136. No clubs, but seminaries, through part of regular course, but not very formal; they are intended to afford students opportunity of working problems under guidance.

143. There are men in each class studying for honors. The principal part of their work is the solution of original problems. I meet them frequently for discussions and suggestions.

158. No clubs; the lectures and recitations regularly extend through an *hour and a half*, and at each original solutions of problems are given, and next time are called for. Each meeting of each class is a seminarium.

Florian Cajori (1890)

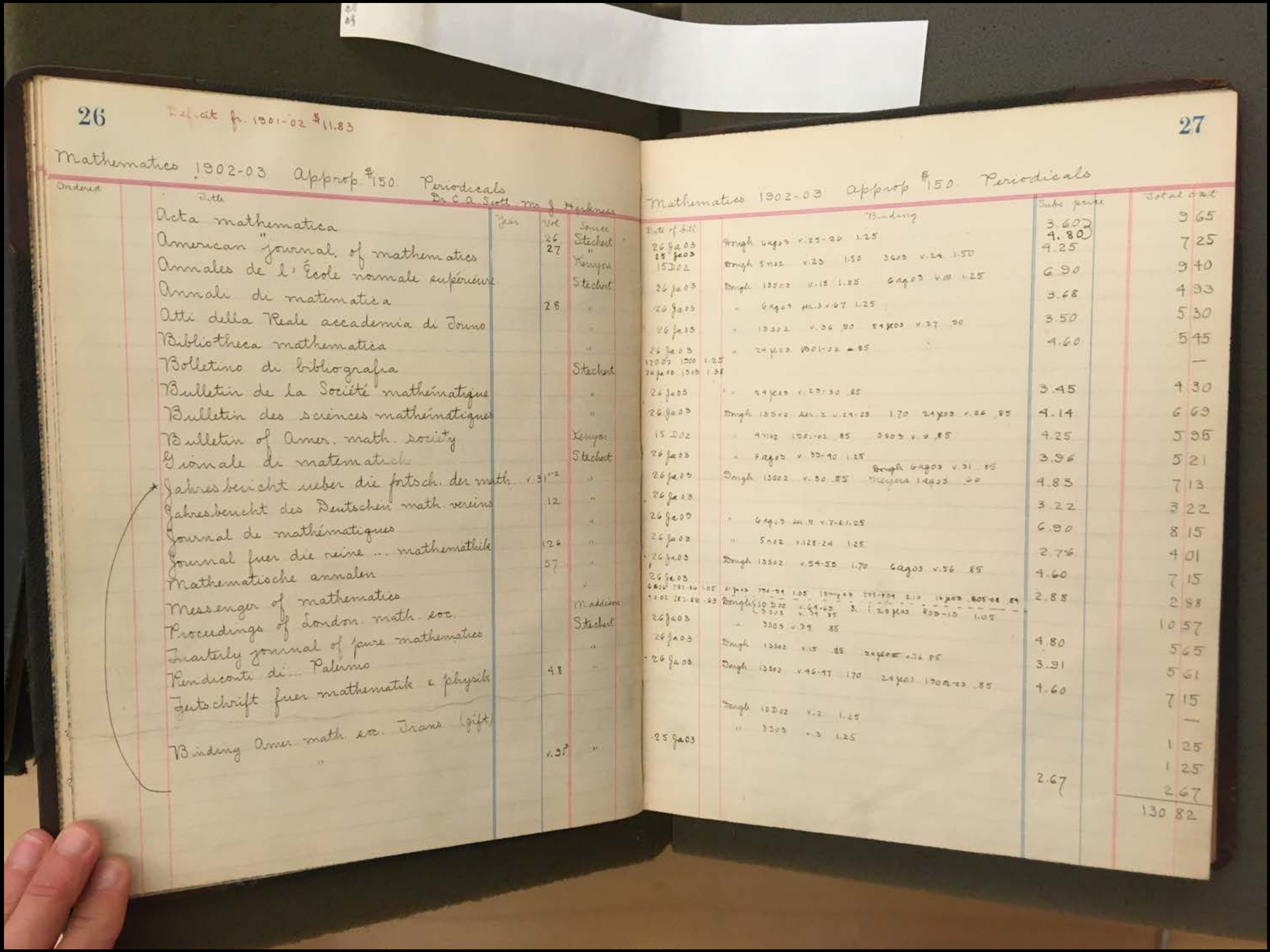
*The teaching and history of mathematics in the United States*

# The Mathematics Journal Club

A. Models

B. Texts

C. Bertrand Russell's 1896 lectures



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The Mathematics Journal Club

A. Models

B. Texts

C. Bertrand Russell's 1896 lectures

One topic that might naturally claim attention at the meetings of the club will be otherwise and more satisfactorily provided for. Ever since the discovery of systems of geometry other than that of Euclid, much attention has been paid to the true foundations of geometrical science, these being discussed in their philosophical as well as their mathematical aspect. No one can for a moment regard the question as settled even yet; and the controversies arising out of the discussions have a far-reaching effect on parts of mathematics that are apparently remote.

Charlotte Angas Scott (1896)
*Annual Report for the President of Bryn Mawr College*

The Mathematics Journal Club

A. Models

B. Texts

C. Bertrand Russell's 1896 lectures

Consequently it is with very special gratification that the mathematical department looks forward to a course of six lectures on the "Foundations of Geometry," to be delivered in November, 1896, by the Hon. Bertrand Russell, Fellow of Trinity College, Cambridge, who comes from England to deliver these lectures in response to the invitation of the Trustees of Bryn Mawr College. These lectures will be on the lines of Mr. Russell's book on the subject, shortly to be published by the Cambridge University Press.

Charlotte Angas Scott (1896)
*Annual Report for the President of Bryn Mawr College*

The Mathematics Journal Club

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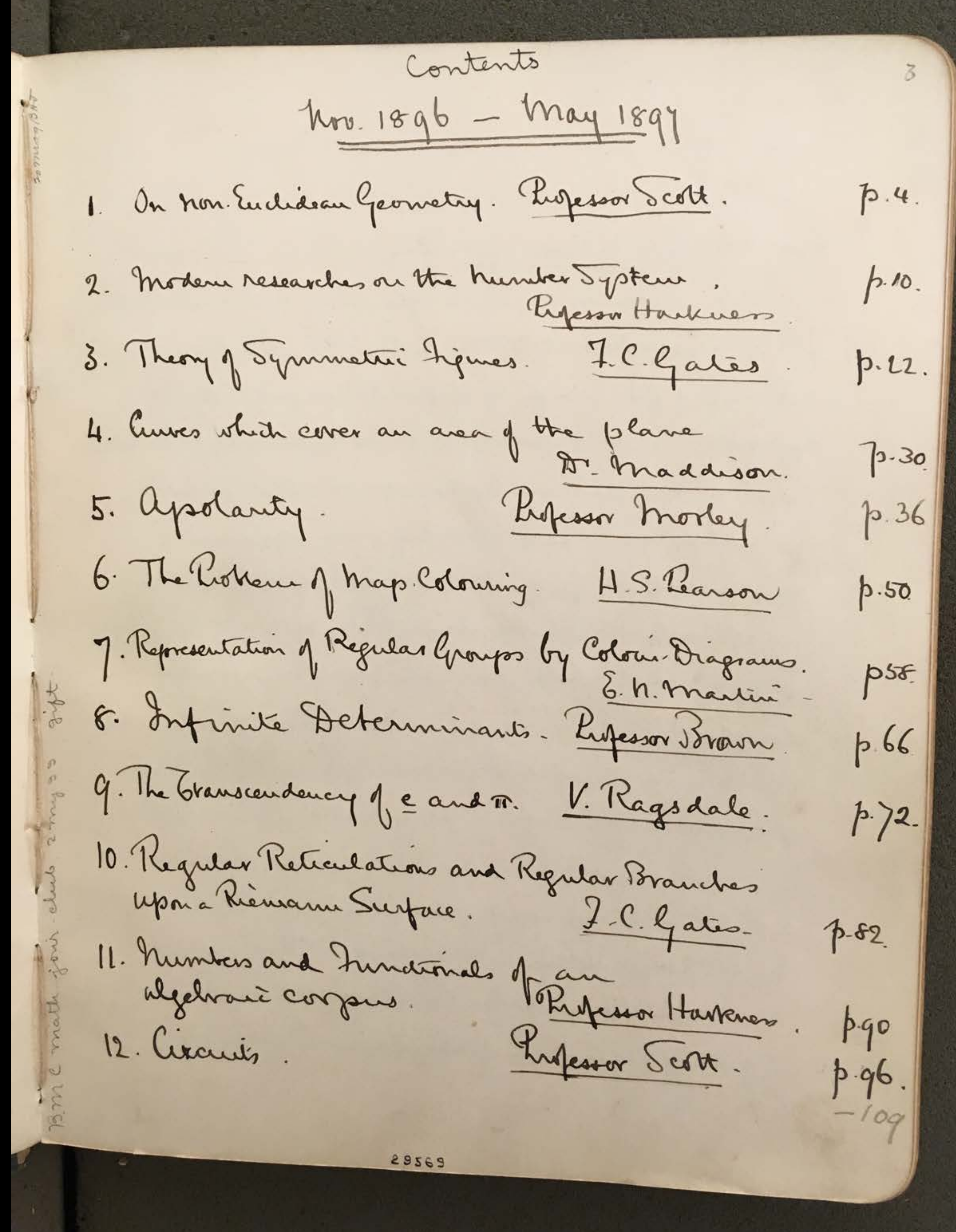
B. Texts

C. Bertrand Russell's 1896 lectures





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Nov. 1896 – May 1897

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7. Conclusions

784.

PRESIDENTIAL ADDRESS TO THE BRITISH ASSOCIATION, SEPTEMBER 1883.

[From the *Report of the British Association for the Advancement of Science*, (1883), pp. 3—37.]

SINCE our last meeting we have been deprived of three of our most distinguished members. The loss by the death of Professor Henry John Stephen Smith is a very grievous one to those who knew and admired and loved him, to his University, and to mathematical science, which he cultivated with such ardour and success. I need hardly recall that the branch of mathematics to which he had specially devoted himself was that most interesting and difficult one, the Theory of Numbers. The immense range of this subject, connected with and ramifying into so many others, is nowhere so well seen as in the series of reports on the progress thereof, brought up unfortunately only to the year 1865, contributed by him to the Reports of the Association; but it will still better appear when to these are united (as will be done in the collected works in course of publication by the Clarendon Press) his other mathematical writings, many of them containing his own further developments of theories referred to in the reports. There have been recently or are being published many such collected editions—Abel, Cauchy, Clifford, Gauss, Green, Jacobi, Lagrange, Maxwell, Riemann, Steiner. Among these the works of Henry Smith will occupy a worthy position.

More recently, General Sir Edward Sabine, K.C.B., for twenty-one years general secretary of the Association, and a trustee, President of the meeting at Belfast in the year 1852, and for many years treasurer and afterwards President of the Royal Society, has been taken from us, at an age exceeding the ordinary age of man. Born October 1788, he entered the Royal Artillery in 1803, and commanded batteries at the siege of Fort Erie in 1814; made magnetic and other observations in Ross and Parry's North Polar exploration in 1818–19, and in a series of other voyages. He

# Imaginaries part I

A. Cayley (1883)

B. Scott (up to 1896)

C. Russell (1897)

[Imaginary space in geometry] has not been, so far as I am aware, a subject of philosophical discussion or enquiry. […] at present, and, considering the prominent position which the notion occupies — say even that the conclusion were that the notion belongs to mere technical mathematics, or has reference to nonentities in regard to which no science is possible, still it seems to me that (as a subject of philosophical discussion) the notion ought to be thus ignored; it should at least be shown that there is a right to ignore it.

Arthur Cayley (1883)

*Presidential Address to the British Association, September 1883*

Imaginaries part I

A. Cayley (1883)

B. Scott (up to 1896)

C. Russell (1897)

In Analytical Geometry we are constrained, as a logical result of the use of coordinates, to consider two fixed imaginary points at infinity, these forming a configuration with reference to which our ideas of parallelism and perpendicularity can be formulated; in Pure Geometry, this configuration does not present itself until we deliberately introduce the conception of imaginary elements, by means of the principle of continuity; but this principle being once admitted, the consideration of this configuration is as inevitable as in the case of Analytical Geometry.

Charlotte Angas Scott (1896)
First Meeting, November 2nd, 1896. On Non-Euclidean Geometry.
*Mathematics Journal Club Notebooks (1896–1897)*

Imaginaries part I

A. Cayley (1883)

B. Scott (up to 1896)

C. Russell (1897)

All space is covered by the range of these three variable quantities: a fresh set of quantities, therefore, such as is introduced by the use of imaginaries, possesses no spatial correlate, and can be supposed to possess one only by a convenient fiction. […] The metaphysician, who should invent anything so preposterous as the circular points, would be hooted from the field. But the mathematician may steal the horse with impunity.

Bertrand Russell (1897)
*An Essay on the Foundations of Geometry*

Imaginaries part I

A. Cayley (1883)

B. Scott (up to 1896)

C. Russell (1897)

The Hon. Bertrand Russell, Fellow of Trinity College, Cambridge, gave a course of six lectures on the Foundations of Geometry by the invitation of the President and Trustees and the Mathematical Department. These lectures were attended by a large number of Bryn Mawr students interested in mathematics and philosophy, and by several representatives of neighboring colleges. The department is under a deep debt of gratitude to Mr. Russell for his interesting and instructive account and criticism of existing theories on metageometry. The lectures were valuable, not merely intrinsically, but also for the stimulus that they have given to the study of geometry.

*The President's Report to the Board of Trustees, for the Year 1896–97*

Imaginaries part I

A. Cayley (1883)

B. Scott (up to 1896)

C. Russell (1897)

Scott's invocation of the principle of continuity to justify imaginaries in pure geometry accords with contemporary English geometers (see Joan Richards, *Mathematical Visions,* 1988).

Russell's strategy dismisses imaginaries as "merely technical" and so "not philosophically valid."

In the years following Russell's lectures at Bryn Mawr, both would revise their treatment of imaginaries.

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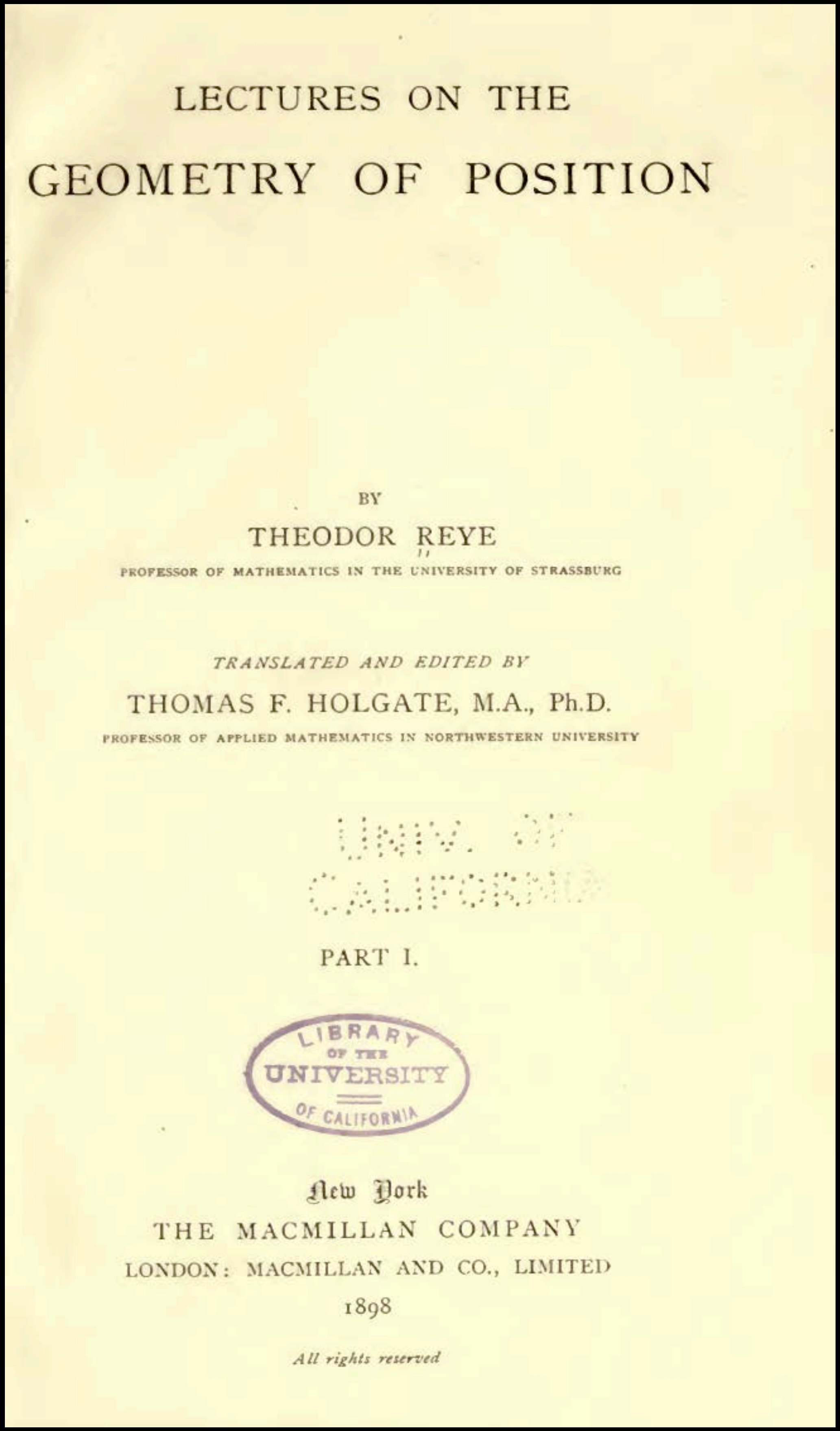
LECTURES ON THE

GEOMETRY OF POSITION

BY

THEODOR REYE

PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF STRASSBURG

*TRANSLATED AND EDITED BY*

THOMAS F. HOLGATE, M.A., Ph.D.

PROFESSOR OF APPLIED MATHEMATICS IN NORTHWESTERN UNIVERSITY



PART I.



New York

THE MACMILLAN COMPANY

LONDON: MACMILLAN AND CO., LIMITED

1898

# Imaginaries part II : Reye and von Staudt

A. Scott reviews Reye
B. A sustained interest
C. Scott's von Staudt
D. Scott's Reye

The lecturer must be prepared to deal with some flagrant logical lapses. Intellectual sincerity forbids that these be passed by without notice; yet I have seen that a frank recognition of their existence shakes a student's faith in the author, and diminishes his interest in the subject.

Charlotte Angas Scott (1898)
Reye's Geometrie der Lage
*Bulletin of the American Mathematical Society*

# Imaginaries part II : Reye and von Staudt

A. Scott reviews Reye

B. A sustained interest

C. Scott's von Staudt

D. Scott's Reye

REYE'S GEOMETRIE DER LAGE.

*Lectures on the Geometry of Position.* By THEODOR REYE, Professor of Mathematics in the University of Strassburg. Translated and edited by THOMAS F. HOLGATE, M.A., PH.D., Professor of Applied Mathematics in Northwestern University. Part I. New York, The Macmillan Company, 1898. 8vo, xix + 248 pp.

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First Meeting; November 23rd 1898.

Von Staudt's treatment of imaginary elements.

THE STATUS OF IMAGINARIES IN PURE GEOMETRY.

(Read before the American Mathematical Society, October 28, 1899.)

THE

MATHEMATICAL GAZETTE.

EDITED BY

W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF

F. S. MACAULAY, M.A., D.Sc.; PROF. H. W. LLOYD-TANNER, M.A., F.R.S.; E. T. WHITTAKER, M.A.

LONDON:

GEORGE BELL & SONS, YORK ST., COVENT GARDEN, AND BOMBAY.

ON VON STAUDT'S GEOMETRIE DER LAGE.

# Imaginaries part II : Reye and von Staudt

A. Scott reviews Reye

B. A sustained interest

C. Scott's von Staudt

D. Scott's Reye

Von Staudt's primary domain is the visible universe; the elements of his geometry, together with the idea of direction, are an intellectual abstraction from the results of observation.

He then extends his domain beyond the visible universe by formal definition; to replace the idea of direction he introduces a set of "ideal points," and finds that the nature of an ideal point is the same as that of a common point, and that the relation of the ideal points and lines to one another is precisely that of points and lines in a plane. Thus his second domain is the visible universe increased by one ideal plane.

Charlotte Angas Scott (1899)
Von Staudt's Treatment of Imaginary Elements
*Mathematics Journal Club Notebook (1898–1899)*

# Imaginaries part II : Reye and von Staudt

A. Scott reviews Reye
B. A sustained interest
C. Scott's von Staudt
D. Scott's Reye

We can, if we choose, picture this ideal plane on any actual plane π, π1 being then pictured on π2, π2 on π3, and so on; there will always be one plane unpictured; however we picture this enlarged universe, it is always "universe + one plane". V. Staudt does not state this very clearly, but it seems to me certain that something of this nature was in his mind, judging from his attitude in later parts of the book.

Charlotte Angas Scott (1898)
First Meeting, November 23rd 1898. Von Staudt's treatment of imaginary elements.
*Mathematics Journal Club Notebooks (1898–1899)*

Imaginaries part II : Reye and von Staudt

A. Scott reviews Reye

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C. Scott's von Staudt

D. Scott's Reye

To attain absolute consistency of language he introduces pairs of feigned, or imaginary, elements. Certain arrangements of pairs of points on a line give rise to a pair of points; certain other arrangements, differing only in their pictured form, apparently do not. Now as we have already extended the domain of geometry beyond the visible universe, the fact that the point pair is not visible cannot be accepted as a proof that it is non-existent. Although the conception of the range of points with its contents is derived from observation, yet we have to take into account the fact that we may not be able to observe the whole.

Charlotte Angas Scott (1899)
The Status of Imaginaries in Pure Geometry
*Bulletin of the American Mathematical Society*

A. Scott reviews Reye

B. A sustained interest

C. Scott's von Staudt

D. Scott's Reye

Hence instead of jumping to the conclusion that the points are non-existent, we investigate the alternative hypothesis, namely, that there are still two points, though not pictured. As regards this purely intellectual hypothesis we have to enquire whether it is tenable, and whether it is manageable.

Of course all this detail fills up page after page, even unto weariness; but by it he does prove that adopting the particular phraseology that assumes the existence of these feigned elements, all verbal consequences are correct.

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Imaginaries part II : Reye and von Staudt

A. Scott reviews Reye

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C. Scott’s von Staudt

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Thus von Staudt, having adjoined to the visible universe a more extensive region, proves that the enlarged domain is a coherent and manageable whole; that there is no essential difference between the elements recognised by the bodily senses and those apprehended by pure intellect.

Charlotte Angas Scott (1900)
On von Staudt’s “Geometrie der Lage” (continued)
*Mathematical Gazette*

Imaginaries part II : Reye and von Staudt

A. Scott reviews Reye

B. A sustained interest

C. Scott's von Staudt

D. Scott's Reye

Thus von Staudt ascends from a visible, actual domain to a more extensive intellectual domain. His work falls naturally into three principal divisions:—

(i.) *intuitive geometry*, dealing with elements regarded as visible entities; this includes the usual propositions of projective geometry, and other investigations such as are now classed as topological (*Geometrie*, pp. 1-23, 30-178; *Beiträge*, pp. 1-76);

(ii.) *constructive geometry*, in which he creates his own enlarged universe by formal definition (*G.* 23-30; *B.* 76-126);

(iii.) *formal geometry*, in which he considers all figures resulting from the combinations of the observed and defined elements (*B.* 131-256).

Charlotte Angas Scott (1900)
On von Staudt's "Geometrie der Lage"
*Mathematical Gazette*

A. Scott reviews Reye

B. A sustained interest

C. Scott's von Staudt

D. Scott's Reye

But in Reye's geometry, where the elements (point, straight line, and plane) are given in an arbitrary manner, without any statement that they visibly exist and without any definition, logic demands a different treatment.

Charlotte Angas Scott (1898)
Reye's Geometrie der Lage
*Bulletin of the American Mathematical Society*

# Imaginaries part II : Reye and von Staudt

A. Scott reviews Reye

B. A sustained interest

C. Scott's von Staudt

D. Scott's Reye

Elements of three kinds are given, absolutely — *a*, *b* and *c*-elements (points, lines, and planes), with a statement of properties and relations to serve instead of definitions. Attaching to these elements the numbers 1, 2, 3, and working with modulus 4, a number of the relations can be obtained arithmetically; for example, 1+1=2 expresses that two points determines line; 3+3 = 6 = 2 [mod.4], two planes determine a line […] A trivial suggestion, perhaps; but it brings out clearly the fact that the elements *a*, *b*, *c* are *not* precisely defined as points, lines, and planes.

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A. Scott reviews Reye

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D. Scott's Reye

Now as I read Reye's first chapter, in the logical development of the system infinitely distant and imaginary elements must be simply a result of classification. We are *given* all points, all lines, all planes; that is all a, b, and c-elements that can have certain relations to one another, with certain properties that hold without exception. Then we *observe* that points, lines, and planes in the visible universe illustrate, or picture, these properties on the whole; and that we can make the picturing more exact, in detail, if we regard parallel lines as meeting somewhere.

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In fact, this is a necessary condition of the picturing; for if two parallel lines do not meet, they cannot represent two *b*-elements that lie in a *c*-element. Thus the conception of infinitely distant elements arises from the attempt to picture this particular intellectual domain in the visible universe. From this attempt there arises also the distinction between real and imaginary elements. For while the visible points, lines, and planes serve to picture some of the *a*, *b*, *c*-elements, we have no right to assume that they suffice to picture all. Allowing for this, we divide elements into picturable and non-picturable and examine whether there are any in the second division.

Charlotte Angas Scott (1899)
The Status of Imaginaries in Pure Geometry
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B. A sustained interest

C. Scott's von Staudt

D. Scott's Reye

Scott acknowledges that she has added her "own interpretation" into these texts.

Von Staudt refers to "unlimited space" [unbegrenzten Raumes] and "representation" [Vorstellung] rather than the "visible universe" and "results of observation."

After chapter I, Reye lacks "the courage of his convictions" to create "a purely logical cold-blooded system, with only intellectual justification or interest."

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In both of these approaches, Scott has reservations with respect to their value in the classroom.

For von Staudt, " the apparent break in passing from the seen to the unseen, […] arouses skepticism as to whether the formal elements can truly be said to 'exist.' While the reason, if sufficiently trained, is convinced, all natural instincts rebel. The whole thing impresses the natural man as simply a tour de force."

For Reye "a purely logical and intellectual geometry" might never "appear to an average student as in any way applicable to anything."

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What of the apparent and the real?

In von Staudt, Scott preserves the linguistic features of ideal and imaginary (feigned) elements as verbal consequences. Hence the apparent and real migrate to a new dichotomy in which "what might be seen" stands in contrast to "what can be said." Scott may have agreed with Russell's 1897 "fiction," but such a designation was not at all grounds for dismissal.

In Reye's system, there is no attention to reality. The geometer may choose what is picturable, but this cannot be everything. So regardless of choice the picture is never complete. But this does not diminish the value of picturing. Instead, Scott finds that this approach "has the additional advantage of justifying the use of diagrams in proving results that depend on so-called imaginaries." Since the choice of what is picturable is arbitrary, a diagrammatic representation is likewise open to multiple interpretations with respect to the pictured elements.

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5. Hilbert's *Grundlagen der Geometrie*
6. Imaginaries part III : all of the above
7. Conclusions

FESTSCHRIFT
ZUR FEIER DER ENTHÜLLUNG DES GAUSS-WEBER-DENKMALS IN GÖTTINGEN. HERAUSGEGEBEN VON DEM FEST-COMITEE. I. THEIL.

GRUNDLAGEN DER GEOMETRIE

VON

DR. DAVID HILBERT,
O. PROFESSOR AN DER UNIVERSITÄT GÖTTINGEN.

MIT 50 IN DEN TEXT GEDRUCKTEN FIGUREN.

LEIPZIG,
VERLAG VON B. G. TEUBNER.
1899.

## Hilbert's *Grundlagen der Geometrie*

A. Not just Hilbert

B. Three problems

C. Poincaré's Review

D. Future research?

Hilbert's work "Grundlagen der Geometrie" (French tr: 1900, English tr: 1902) was described in the Bulletin of the American Mathematical Society, vol. 6, 1900 by Dr. Sommer of Göttingen. It has been discussed and criticised by (among others) Schur (M. Annalen vol. 55, 1901) and E. H. Moore (Tr. A. M. Soc. vol. 3 1902). An exceedingly interesting critical analysis by Poincaré appeared in Darboux' "Bulletin" for September 1902. In the "Questioni riguardante la Geometria elementare" of F. Enriques the first twenty pages should be read in this connection.

Charlotte Angas Scott (1902)

Second Meeting, November 25th, 1902. Hilbert's Foundations of Geometry.

*Mathematics Journal Club Notebooks (1902–1903)*

Hilbert's *Grundlagen der Geometrie*

A. Not just Hilbert

B. Three problems (after Enriques)

C. Poincaré's Review

D. Future research?

As regards the Foundations of Geometry, there are three distinct problems.

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# Hilbert's *Grundlagen der Geometrie*

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There is (1) the psychological question as to the acquisition of spatial notions;

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(2) the philosophical inquiry whether geometry rests on an empirical basis; if this be so, then the propositions of Geometry, like those of Physics, have a purely relative validity. The inquiry of the physicist "what do we know"? Leads to the inquiry of the metaphysician "do we know anything?". In this sense, the mathematician does not wish to "know". Mathematical certainty demands that the reasoning be based on a frank hypothesis: — if such and such assumptions be made, then such and such conclusions follow. Thus as regards the Foundations of G…

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(3) the mathematical problem is purely logical. Not "what premisses does experience offer us?" Not "how did we arrive at these premisses"? But simply "what are the necessary premisses?"

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Taking the problem in this sense, by the Foundations of Geometry we understand (1) the set of primitive concepts (elements) and logical concepts (possibilities of combination etc.), together with (2) the set of postulates and axioms that express the laws of the appropriate logic. The concepts ought, so far as possible, to be independent; but the most important thing is that they be general. It is necessary that it be possible to ignore the content (express by pure symbols), as otherwise there is the danger that we may unconsciously rely on unexpressed conclusions of intuition.

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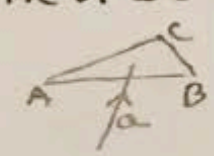
# Synopsis of Hilberts system of Axioms.

| | I. Combination | II Arrangement. | III. | IV Congruence | V. |
|---|---|---|---|---|---|
| Linear | | 1. If ABC, then CBA.<br>2. Given A, C, there exists ABC, also ACD.<br>3. Given A.B.C. on line, then either BAC, ABC, or ACB<br>4. x | | 1. Congruent segments exist.<br>2. If AB ≡ A'B' and AB ≡ A''B'' then A'B' ≡ A''B''.<br>3. If AB, BC ≡ A'B', B'C', then AC ≡ A'C'. | 1. axiom of Archimedes |
| Plane | 1. Two pnts. determine line<br>2. Any two ^of a line^ pts. det. ^that^ line<br>7.1. On any line there exist at least two points. | 5. If a cuts AB, it must cut either AC or BC.<br><br> | 1. Parallel axiom. (of Euclid) | 4. Congruent angles<br>5. If hk ≡ h'k', and hk ≡ h''k'' then h'k' ≡ h''k''.<br>6. If AB, AC, ∠BAC ≡ A'B' etc., then ∠s B, C ≡ B', C'. | |
| Space | 3. Three pnts. det: plane<br>4. Any three ^of a pl. & non-coll.^ pts. det. ^that^ plane.<br>5. If two points on plane, line on plane<br>6. If two planes have one common pt, then another.<br>7.2, 3. On any plane at least 3 pts, non-coll. In space at least 4 pts non-in pl. | | | | |

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From the primitive concepts with the help of the postulates (axioms) other definable concepts may be derived, which logically might equally well have served as primitive concepts. The mathematician can judge between these only by comparing the development of the argument according as one set or another is adopted. For him there can be no à priori ground for preferring one to another; the preference is based on à posteriori evidence.

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The philosopher, on the contrary, feels that there must be some external reason for preferring one to another. Thus Poincaré, (whose attitude on this question is the philosophical one) criticises Hilbert on this very point : "The axioms are there, no one knows whence they came, and so it is just as easy to lay down A as B".

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A specially interesting criticism of Poincaré's relates to the Groups I and II. The axioms of II are presented as depending on I; nevertheless in Analysis Situs the axioms II are needed, although the subject knows nothing of the concepts used in I (line and surface are the concepts required, not straight line and plane). Could not II be enunciated independently of I? At present, Analysis Situs relies on pure intuition; the problem of its logical foundation is hitherto untouched.

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> The fact that each group of axioms can be dispensed with, logically, is an obvious challenge to construct as much of the ordinary geometry as possible without using each group.

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Hilbert's *Grundlagen* is the "logical cold-blooded" realization of what Scott had hoped Reye might be. The consequent elimination of reality and appearance – so central to Scott's research in analytical geometry – does not appear of concern in her approbation.

Scott's shift in interest reflects a broader disciplinary shift. In a passage not cited by Scott, Poincaré warned that "the mind revolts against conceptions" such as non-Archimedean magnitudes and explained that this was because "through an old habit, it is looking for a visual image." Scott, along with many geometers of her era, freed herself "from this prejudice."

However, there were other geometries in which the visual remained important.

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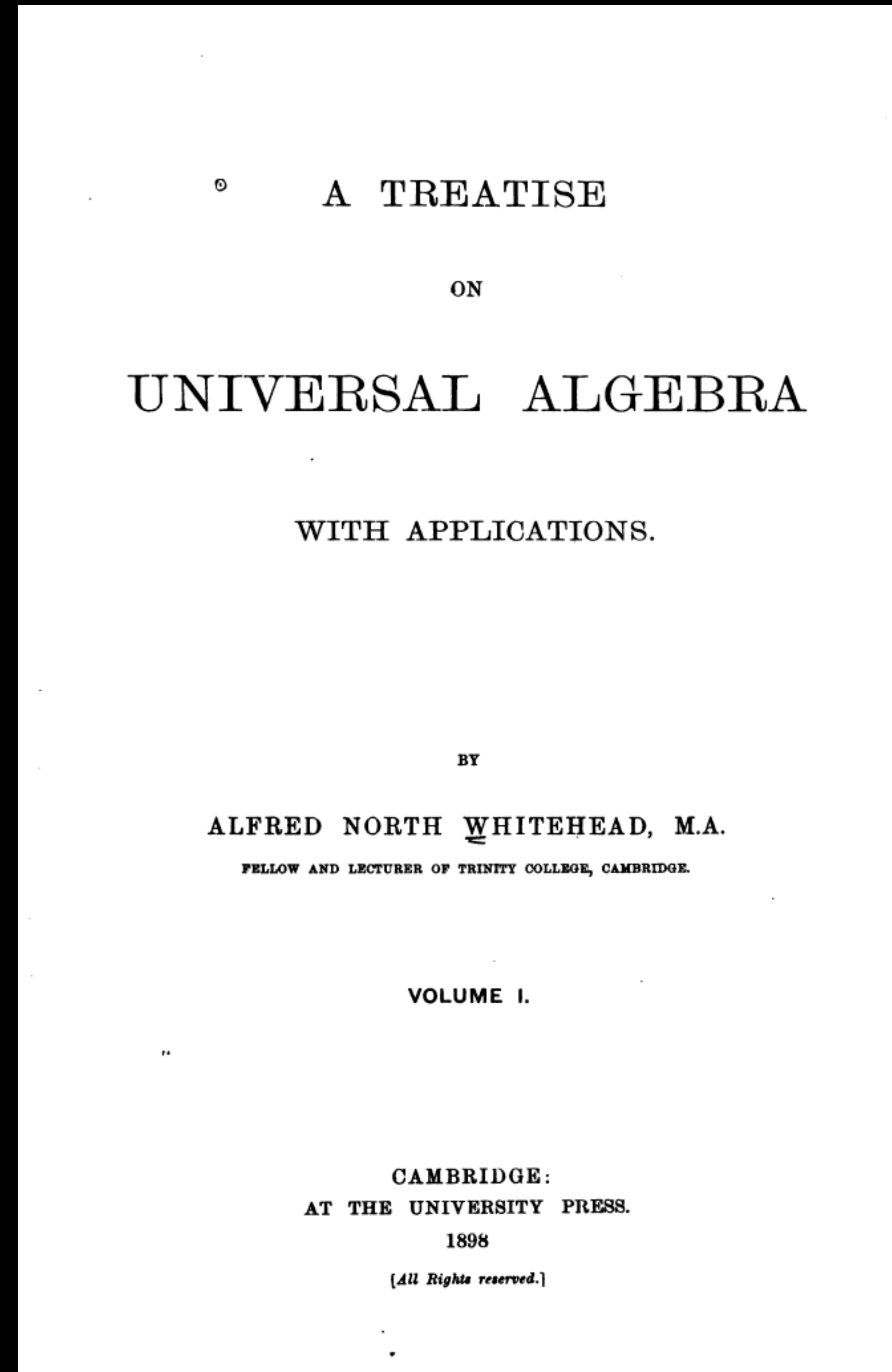

A TREATISE

ON

UNIVERSAL ALGEBRA

WITH APPLICATIONS.

BY

ALFRED NORTH WHITEHEAD, M.A.

FELLOW AND LECTURER OF TRINITY COLLEGE, CAMBRIDGE.

VOLUME I.

CAMBRIDGE:
AT THE UNIVERSITY PRESS.
1898

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## Imaginaries part III : all of the above

A. Whitehead's *Universal Algebra*

B. Question of fact

C. Question of hypothesis

D. An illustration

The individuals about which we reason in Mathematics are in general signs or symbols. Signs have been divided into (1) suggestive signs, (2) expressive signs, (3) substitutive sings; […] mathematical symbols are substitutive signs. In the use of an expressive sign attention is fixed on the meaning; but a substitutive sign takes the place of that for which it is substituted. (See Whitehead, Universal Algebra, Ch. I.)

Charlotte Angas Scott (1902)
First Meeting, November 25th, 1903. On Imaginary Quantities.
*Mathematics Journal Club Notebooks (1903–1904)*

Imaginaries part III : all of the above

A. Whitehead's *Universal Algebra*

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The difference between words and substitutive signs has been stated thus, 'a word is an instrument for thinking about the meaning which it expresses; a substitute sign is a means of not thinking about the meaning which it symbolizes*.' The use of substitutive signs in reasoning is to economize thought.

* Cf. Stout, "Thought and Language," *Mind*, April 1891.

Alfred North Whitehead (1898)
*A Treatise on Universal Algebra with Applications.*

Imaginaries part III : all of the above

A. Whitehead's *Universal Algebra*

B. Question of fact

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D. An illustration

A great part of mathematics is therefore simply the representation of the quantities by appropriate symbols. The representation of any set of elements (quantities, operations, etc) by another set, one with whose relations we are more familiar, is serviceable only if a sufficient number of the relations of the symbolic set can be applied to the original set. The advantage is the economy of intellectual effort, in that we are enabled to utilise results already obtained; the danger is that we may unconsciously credit the original set with the properties of the symbolic set.

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A representation, otherwise good, may err in excess or defect. If in excess, we may confine ourselves to a part. […] If the representation is in defect, a part only of the field is represented, the rest is left to the imagination. The elements of the original field are divided into two classes, (i) the represented, (ii) the <u>non-represented</u> or <u>imaginary</u>.

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where we start with the field and a particular representation, the question as to imaginaries is one of fact; do the symbols exhaust the elements?

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There is however an alternative intellectual process. We may start with the representation; we may possibly in the beginning be under the impression that we have the whole field. But (by analogy with other fields) we may be led to adopt the hypothesis that there are other, non-represented, elements; that is, imaginaries.

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when we begin with the representation, the question is one of hypothesis; in general we have the choice of alternative hypotheses, (i) the field is not more extensive than the representation, (ii) the field is more extensive than the representation.

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First Meeting, November 25th, 1903. On Imaginary Quantities.
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Imaginaries part III : all of the above

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Even in Algebra we have this choice, and we accept the second hypothesis, introducing imaginaries in order to provide equations with solutions. "There is, however, no law of nature to the effect that every equation <u>must</u> have a root." (Russell, Principles of Mathematics, I. p. 378) and we are at liberty to adopt the other hypothesis.

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But in the development of the theory, there comes a stage when from a set of data sometimes a result is obtained, and sometimes not, as for example, in the determination of a pair of points that shall be harmonic conjugates with respect to each of two given pairs; sometimes a pair can be found, sometimes not. If the eye alone were concerned, this variation in results would not offend, for there is a difference in the appearance of the two cases. But the "algebraic" instinct — or the pure intellect — (reveling in sameness) objects, even revolts, and insists on absolute equality in consequences. Looking upon the points as mere symbols of some unspecified original, it finds its refuge in the suggestion that possibly the representation is imperfect, incomplete.

# Imaginaries part III : all of the above

A. Whitehead's *Universal Algebra*

B. Question of fact

C. Question of hypothesis

D. An illustration

If it be both tenable and manageable, then the hypothesis may be entertained; it is open to us to consider the more extensive field, for which the representation is only partial, thus leaving imaginaries, unrepresented elements.

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First Meeting, November 25th, 1903. On Imaginary Quantities.

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C. Question of hypothesis

D. An illustration

Von Staudt shows that any such series of points (a strand) can be projected into the real points of a real straight line, and that from any one strand, thus represented, all the rest can be inferred. The concept of "betweenness", and the Axiom of Archimedes (see Hilbert's Grundlagen der Geometrie) are seen to belong to each separate strand of the straight line, though not to the line as a whole.

Charlotte Angas Scott (1902)

First Meeting, November 25th, 1903. On Imaginary Quantities.

*Mathematics Journal Club Notebooks (1903–1904)*

# Imaginaries part III : all of the above

A. Whitehead's *Universal Algebra*

B. Question of fact

C. Question of hypothesis

D. An illustration

In the broader context of mathematical signs, the graphic appearance of geometric objects and relations is an instance of a choice of representation. Accordingly what is "apparent" is a question of fit between fields. The choice of representation offers the potential for "aid to the imagination in the process of reasoning" as Scott quoted from Whitehead.

He further emphasized the reciprocity between the two fields: "No sooner has a substitutive scheme been devised to assist in the investigation of any originals, than the imagination begins to use the originals to assist in the investigation of the substitutive scheme." For this reason, Whitehead suggests "it would be better to abandon the conception of originals studied by the aid of substitutive schemes, and to conceive of two sets of inter-related things studied together, each scheme exemplifying the operation of the same general laws."

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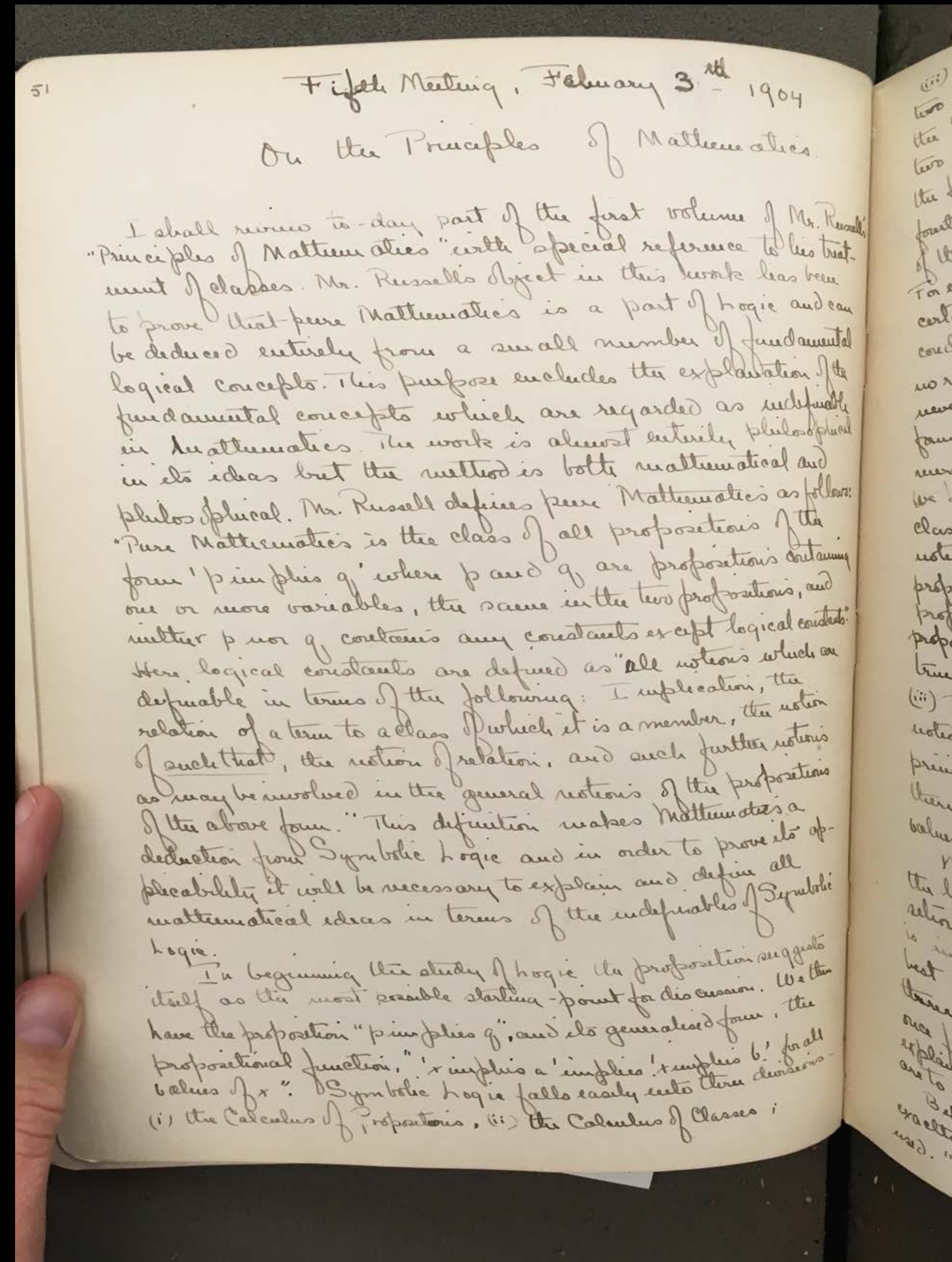
Such a perspective aligns with Scott's dedication to analytical geometry, in which the figure and the equation are complementary modes of investigation and discovery. At this level, Scott's writing on the alignment between real and imaginary elements parallels her work on the alignment between the geometrical subject and the algebraic language of expression. In both cases, the correspondence leads to greater geometrical understanding.

1. Charlotte Angas Scott and Bryn Mawr College
2. The Mathematics Journal Club
3. Imaginaries part I : Cayley, Scott, Russell
4. Imaginaries part II : Reye and von Staudt
5. Hilbert's *Grundlagen der Geometrie*
6. Imaginaries part III : all of the above
7. Conclusions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Fifth Meeting, February 3th 1904

On the Principles of Mathematics

I shall review to-day part of the first volume of Mr. Russell's "Principles of Mathematics" with special reference to his treatment of classes. Mr. Russell's object in this work has been to prove that pure Mathematics is a part of Logic and can be deduced entirely from a small number of fundamental logical concepts. This purpose excludes the explanation of the fundamental concepts which are regarded as indefinable in Mathematics. The work is almost entirely philosophical in its ideas but the method is both mathematical and philosophical. Mr. Russell defines pure Mathematics as follows: "Pure Mathematics is the class of all propositions of the form 'p implies q' where p and q are propositions containing one or more variables, the same in the two propositions, and neither p nor q contains any constants except logical constants." Here logical constants are defined as "all notions which are definable in terms of the following: Implication, the relation of a term to a class of which it is a member, the notion of such that, the notion of relation, and such further notions as may be involved in the general notions of the propositions of the above form." This definition makes Mathematics a deduction from Symbolic Logic and in order to prove its applicability it will be necessary to explain and define all mathematical ideas in terms of the indefinables of Symbolic Logic.

In beginning the study of Logic the proposition suggests itself as the most possible starting-point for discussion. We then have the proposition "p implies q", and its generalised form, the propositional function, "'x implies a' implies 'x implies b' for all values of x". Symbolic Logic falls easily into three divisions — (i) the Calculus of Propositions, (ii) the Calculus of Classes;

## Conclusions

A. Iconic texts shaping local knowledge traditions

B. Pedagogy and philosophy

In the midst of Scott's efforts to better communicate the role of imaginaries in pure geometry, pure geometry was moving in different directions. Scott's progression from von Staudt to Hilbert echoes a broader trend and demonstrates on a micro-scale the evolution within "normal science" toward new definitions of geometry. In the shift away from the "visible universe," Scott shows no sign of unease or anxiety.

The characteristics of Scott's approach to analytical geometry may not have served to generate new research within pure geometry, but this did not diminish her ability to enthusiastically communicate both studies. Through this simultaneous engagement, Scott could situate her attention to appearance as instantiating "the representation of the quantities by appropriate symbols."

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Conclusions

A. Iconic texts shaped by local knowledge traditions
B. Pedagogy and philosophy

The expense and rarity of foreign titles for Bryn Mawr students (and students in the United States more generally) suggests that the Journal Club served as a first introduction to certain books and topics.

In the Journal Club von Staudt "observes" and "adjoins," Reye distinguishes between the picturable and unpicturable, and Hilbert is accompanied by Poincaré as interlocutor.

Scott's gatekeeper position is especially significant in this context because one student in the audience, Marion Reilly, would go on to pursue, though never complete, a doctorate that is vaguely referenced as at the intersection of geometry and philosophy. In the same year that Scott spoke on Imaginary Quantities, Reilly delivered a talk "On the Principles of Mathematics" --- a review of Russell's text.

Reilly spent spring 1907 in Cambridge where she worked with Russell and attended the lectures of Whitehead and J. J. Thomson.

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In bringing together Scott's published expository texts and her unpublished Journal Club entries, I have somewhat muddled the distinction of audiences.

Scott's publications are pedagogically oriented and she displays a concern for the method of the class-room as well as the reactions of the "natural man." This conservatism does *not* inform Scott's Journal Club entries, which attend to the feelings of "pure intellect" who "insists on absolute equality in consequences."

Would Scott consider these latter writings philosophical?

Following her own tripartite division, her writings on the foundations of geometry go beyond the "purely logical" mathematical question. But the line between disciplines, like the line between what subjects remained of interest, seems deeply personal. In any case, Scott's direct dialogue with acknowledged philosophers remained restricted to the intimate audience of the Mathematical Journal Club.

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Date 12 January 1925

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13 Jan. 1925

thank you.

We may if we choose concentrate our attention entirely on the laws and operations manifested in these different geometrical figures; these figures with their related theories are then unimportant transitory incarnations of an underlying unchangeable principle; the differences perceived by the eye are neglected and the figures are regarded as the same inasmuch as they express the same sequence and connection of elements. From this point of view a curve and all its projections, or a curve and its reciprocal, are essentially identical; a system of lines in a plane may be identical with a system of conics in a net. […]

We are however here concerned with the manifestations of the underlying principles and operations; these last exist for us only as the cause of the correspondence that we consider.

Charlotte Angas Scott (1894)
*An Introductory Account of Certain Modern Ideas and Methods in Plane Analytical Geometry*

Imaginaries part I

A. Cayley (1883)

B. Scott (up to 1896)

C. Russell (1897)

Imaginaries present themselves naturally in the solution of algebraic equations, and are then recognized for the sake of continuity. If now we refuse to admit them into algebraic geometry, we shall have to examine the work at every step, to see whether it has a legitimate application in geometry; our symbolical language will no longer have an exact relation to the subject matter.

Note. The introduction of imaginary elements into Pure Geometry depends on different considerations, and requires independent justification.

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Imaginaries part I

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All attempts to construct imaginaries have been wholly abandoned in pure geometry; but, by asserting once for all the principle of continuity as universally applicable to all the properties of figured space, geometers have succeeded, if not in explaining the nature of imaginaries, yet, at least, in deriving from them great advantages. They consider it a consequence of the law of continuity that if we once demonstrate a property for any figure in any one of its general states, and if we then suppose the figure to change its form, subject of course to the conditions in accordance with which it was first traced the property we have proved, though it may become unmeaning, can never become untrue, even if every point and every line by means of which it was originally proved should disappear.

H. J. S. Smith (1851)
On Some of the Methods at Present in Use in Pure Geometry
*Transactions of the Ashmolean Society*

# Imaginaries part III : all of the above

A. Whitehead's *Universal Algebra*

B. Question of fact

C. Question of hypothesis

D. An illustration

The introduction of imaginary elements into the field of investigation of Pure Geometry illustrates the second process, that in which we begin with the representation. The straight line (with points as elements) as used in elementary geometry, is a given field, represented in a diagram. The theory is simple enough; the proofs rely on certain properties of the straight line, in great measure the results of observation: — as, e.g. the concept implied in the word "between", used to express relative position of points of the line. The intellect accepts these conclusions of the eye; the straight line, as understood by the intellect, <u>is</u> represented by the straight line, as seen by the eye.

Hilbert's *Grundlagen der Geometrie*

A. Not just Hilbert

B. Three problems (after Enriques)

C. Poincaré's Review

D. Future research?

No meaning must be attached to the word "determine"; simply that in some unknown manner for every pair AB there is a p. Similarly for such words as "order" "between" and "congruence"

Charlotte Angas Scott (1902)
Second Meeting, November 25th, 1902. Hilbert's Foundations of Geometry.
*Mathematics Journal Club Notebooks (1902–1903)*

Imaginaries part I

A. Cayley (1883)
B. Scott (up to 1896)
C. Russell (1897)

That the notion of imaginary points is of supreme importance in Geometry, will be seen by any one who reflects that the circular points are imaginary, and that the reduction of metrical to projective Geometry, which is one of Cayley's greatest achievements, depends on these points. But to discuss adequately their philosophical import is difficult to me, since I am unacquainted with any satisfactory philosophy of imaginaries in pure Algebra. I will therefore adopt the most favourable hypothesis, and assume that no objection can be successfully urged against this use. Even on this hypothesis, I think, no case can be made out for imaginary points in Geometry.

Bertrand Russell (1897)
*An Essay on the Foundations of Geometry*

An epistemic configuration of mathematical research is the entirety of the intellectual resources that are involved in a particular research episode. It comprises the mathematical language, the skills and techniques at the disposal of the mathematician or the group of mathematicians engaged in this research, the set of research topics and open problems under consideration, the horizon of aims and more general heuristic guidelines followed by the researchers, etc.

Moritz Epple (2006)
Knot Invariants in Vienna and Princeton during the 1920s:
Epistemic Configurations of Mathematical Research
*Science in Context*